FAMAT Calculus Individual Test Jan 2003 Answers

1. C
2. C
3. C
4. C
5. C
6. C
7. C
8. E
9. D
10. E
11. C
12. C
13. B
14. D
15. B
16. E
17. B
18. D
19. A
20. D
21. A
22. D
23. D
24. E
25. D
26. D
27. E
28. D
29. D
30. D

1. $f(x)=\sin^2 x \Rightarrow f'(x)=2\sin x\cos x=\sin 2x$ So $f'(2x)=\sin 4x$ and since $\sin 4x=0 \Rightarrow 4x=0, 4x=\pi, 4x=2\pi, \ldots$ we have
$\sin 4x=0 \Rightarrow x=0, x=\frac{\pi}{4}, x=\frac{\pi}{2}, \ldots$. The answer is **C**.

2. $\int_{-2}^{2}(4-x^2)dx=\left(8-\frac{8}{3}\right)-\left(-8+\frac{8}{3}\right)=\frac{32}{3}$. The line $y=k$ will intersect this curve at $x=\pm\sqrt{k}$. So $\int_{-\sqrt{k}}^{\sqrt{k}}(k-x^2)dx=\frac{16}{3}$ which means $\int_{0}^{\sqrt{k}}(k-x^2)dx=\frac{8}{3}$ since this is an even function. This equation is easier to solve.
$k\sqrt{k}-\frac{k\sqrt{k}}{3}=\frac{8}{3} \Rightarrow \frac{2k\sqrt{k}}{3}=\frac{8}{3} \Rightarrow k\sqrt{k}=4 \Rightarrow k=\sqrt[3]{16}$. The answer is **C**.

3. $f(x)=x^n \Rightarrow f'(x)=nx^{n-1} \Rightarrow f''(x)=n(n-1)x^{n-2}$. Extending this argument to the nth derivative yields $n(n-1)(n-2)(n-3)\ldots 1\bullet x^0=n!$ The answer is **C**.

4. $\int_{-a}^{0}f(x)dx=N$ since this is an odd function it must also be true that
$\int_{0}^{a}f(x)dx=N$. Unfortunately symmetry provides us with no clues about
$\int_{0}^{2a}f(x)dx$ so the answer is **C**.

5. $2x-3y=6 \Rightarrow 2x-6=3y \Rightarrow \frac{2}{3}x-2=y$. $f'(x)=\frac{3}{2\sqrt{3x+1}}$ and we want to know when this is equal to $\frac{2}{3}$.
$\frac{2}{3}=\frac{3}{2\sqrt{3x+1}} \Rightarrow \frac{9}{4}=\sqrt{3x+1} \Rightarrow \frac{65}{48}=x$ The answer is **C**.

6. $\int_{0}^{\pi}(\sin^2 x+\cos^2 x)dx=\int_{0}^{\pi}1dx=\pi-0=\pi$ The answer is **C**

7. The Mean Value Theorem says that $f'(c)=\frac{f(b)-f(a)}{b-a}$ so for this function we have $\frac{2}{\sqrt{c}}+3=\frac{20-7}{3} \Rightarrow \frac{2}{\sqrt{c}}=\frac{4}{3} \Rightarrow 6=4\sqrt{c} \Rightarrow \frac{9}{4}=c$. The answer is **C**.

8. $\lim\limits_{h\to 0}\dfrac{\cot\left(\dfrac{\pi}{4}+h\right)-1}{h}$ is the definition of the derivative of $f(x)=\cot x$ at $x=\dfrac{\pi}{4}$. Since $f'(x)=-\cot x\csc x$, $f'\left(\dfrac{\pi}{4}\right)=-1\bullet\sqrt{2}=-\sqrt{2}$. The answer is **E**

9. $y=3x^{-2}\Rightarrow\dfrac{dy}{dx}=-6x^{-3}\Rightarrow\dfrac{dy}{dx}=\dfrac{6}{x^3}$ so at $x=2$, $\dfrac{dy}{dx}=\dfrac{-6}{8}=\dfrac{-3}{4}$ So we know that the normal slope is $\dfrac{4}{3}$. At $x=2$, $y=\dfrac{3}{4}$ so the normal line equation is $y-\dfrac{3}{4}=\dfrac{4}{3}(x-2)$. The answer is **D**.

10. $\lim\limits_{h\to 0}\dfrac{\sin^2(5x)}{2x^2}=\dfrac{1}{2}\left[\lim\limits_{h\to 0}\dfrac{\sin 5x}{x}\bullet\lim\limits_{h\to 0}\dfrac{\sin 5x}{x}\right]=\dfrac{1}{2}\bullet 5\bullet 5=\dfrac{25}{2}$ The answer is **E**

11. $f(x)=\dfrac{x^3-2x}{x^2-4}=\dfrac{x\left(x^2-2\right)}{(x+2)(x-2)}$ This function has two horizontal asymptotes and no vertical asymptotes. The answer is **C**

12. $V=\dfrac{4}{3}\pi r^3\Rightarrow\dfrac{dv}{dt}=4\pi r^2\dfrac{dr}{dt}\Rightarrow 3=4\pi r^2\dfrac{dr}{dt}$ but we need to know something about the radius here. Since $V=36\pi$ we can see that $r=3$. So we now have $3=4\pi r^2\dfrac{dr}{dt}\Rightarrow 3=4\pi 3^2\dfrac{dr}{dt}\Rightarrow\dfrac{1}{12\pi}=\dfrac{dr}{dt}$ Look at formula for surface area, $SA=4\pi r^2\Rightarrow\dfrac{dSA}{dt}=8\pi r\dfrac{dr}{dt}\Rightarrow\dfrac{dSA}{dt}=2$ The answer is **C**

13.

$$\frac{\dfrac{1}{x+h+1}-\dfrac{1}{x+1}}{h}=\frac{\dfrac{x+1-x-h-1}{(x+1)(x+h+1)}}{h}=\frac{-h}{h(x+1)(x+h+1)}=\frac{-1}{(x+1)(x+h+1)}$$

The answer is **B**

14. $\lim\limits_{h\to 0}\dfrac{1-\cos^2 5h}{h}=\lim\limits_{h\to 0}\dfrac{\sin^2 5h}{h}=\lim\limits_{h\to 0}\dfrac{\sin 5h}{h}\bullet\lim\limits_{h\to 0}\sin 5h=5\bullet 0=0$ The answer is **D**

15. $0.4(1)+k^2=k+2.4\Rightarrow k^2-k-2=0\Rightarrow(k-2)(k+1)=0$ So our two solutions are $k=2$, $k=-1$. The answer is **B**

16. When is it true that $f'(x)=\sin(x^2)$ is greater than zero? Since $\sin\alpha>0$ on $(0,\pi)$ we see that $0<x^2<\pi\Rightarrow 0<x<\sqrt{\pi}$ it is also true that $0<x^2<\pi\Rightarrow -\sqrt{\pi}<x<0$ solves this, but at $x=0$ the function is not increasing so the answer is **E**

17. $f(x)=x^2-x^{-1/2}\Rightarrow f'(x)=2x+\frac{1}{2}x^{-3/2}\Rightarrow f''(x)=2-\frac{3}{4}x^{-5/2}$ Since we want the second derivative to equal zero we have $0=2-\frac{3}{4}x^{-5/2}\Rightarrow 2=\frac{3}{4\sqrt{x^5}}$

This means that $\sqrt{x^5}=\frac{3}{8}\Rightarrow x^5=\left(\frac{3}{8}\right)^2\Rightarrow x=\left(\frac{3}{8}\right)^{2/5}$ The answer is **B**

18. $s(t)=2t^3-9t^2+5\Rightarrow v(t)=6t^2-18t$ Since $a(t)=12t-18$ the $6t^2-18t=0\Rightarrow t=3$ acceleration at the time value we're interested in is 18. The answer is **D**

19. $y=4-x^{2/3}$ and each square has side length of y so we have

$$\int_0^8 y^2dy=\int_0^8\left(4-x^{2/3}\right)^2dx=\int_0^8\left(16-8x^{2/3}+x^{4/3}\right)dx$$ The answer is **A**

20. $y=f^3(x)\Rightarrow \frac{dy}{dx}=3f^2(x)\bullet f'(x)\Rightarrow$

$\frac{d^2y}{dx^2}=6f(x)\bullet\left(f'(x)\right)^2+3f^2(x)\bullet f''(x)$ At $x=2$ we have

$\frac{d^2y}{dx^2}=6(6)(4^2)+3(6^2)(2)=792$ The answer is **D**

21. $f(x)=ax^4+bx^2\Rightarrow f'(x)=4ax^3+2bx$ This shows that the derivative is equal to zero when $x=0$ or $2ax^2+b=0\Rightarrow x=\pm\sqrt{\frac{-b}{2a}}$ The answer is **A**

22. $f(x)=-x^3+Ax^2+Bx+30\Rightarrow f'(x)=-3x^2+2Ax+B$
$\Rightarrow f''(x)=-6x+2A\Rightarrow f''(4)=0=-24+2A\Rightarrow A=12$ Since $f'(x)=0$ for $x=-2$ and $x=10$ we can use
$f'(10)=0=-300+240+B\Rightarrow 60=B$ The answer is **D**

23. The average rate of change of thefunction is the slope of the secant line for the region. The answer is **D**

24. The only information we have is that the function exists at the point (2,3). The answer is **E**

25. If f is continuous then $k=\dfrac{x^3-8}{x-2}$ for $x\to 2$

$k=\dfrac{(x-2)(x^2+2x+4)}{x-2}\Rightarrow k=x^2+2x+4$ As $x\to 2,\ k\to 12$ The answer is **D**

26. $x^2+3y^2=4\Rightarrow 2x+6y\dfrac{dy}{dx}=0\Rightarrow\dfrac{dy}{dx}=\dfrac{-2x}{6y}$ At the point (1,1) we have

$\dfrac{dy}{dx}=\dfrac{-1}{3}$. The answer is **D**

27. Perimeter of the region is $2x+y$ and we know that $xy=20000$. Substituting gives us $P(x)=2x+\dfrac{20000}{x}\Rightarrow P'(x)=2-\dfrac{20000}{x^2}$ We want the derivative to equal zero so $2=\dfrac{20000}{x^2}\Rightarrow 2x^2=20000\Rightarrow x=100$ So, perimeter is 400 and the answer is **E**

28. Call the original amount P. At time = 8 we have

$\dfrac{P}{2}=Pe^{8r}\Rightarrow\dfrac{1}{2}=e^{8r}\Rightarrow\dfrac{-\ln 2}{8}=r$ When is there one third of the original amount remaining? $\dfrac{P}{3}=Pe^{tr}\Rightarrow\dfrac{1}{3}=e^{tr}\Rightarrow\dfrac{-\ln 3}{r}=t$. The answer is **D**

29. $V=\pi r^2h\Rightarrow\dfrac{dV}{dt}=2\pi rh\dfrac{dr}{dt}+\pi r^2\dfrac{dh}{dt}$ Since radius doesn't change, when $h=10$ we have $\dfrac{dV}{dt}=160\pi$ The answer is **D**

30. $A_1=\dfrac{1}{2}\left(\dfrac{1}{2}\right)\left(\sin 1+\sin\sqrt{1.5}\right)$ and $A_2=\dfrac{1}{2}\left(\dfrac{1}{2}\right)\left(\sin\sqrt{1.5}+\sin\sqrt{\ }\right)$ The answer is **D**