4

CALCULUS INDIVIDUAL TEST
January
Answers to Individual Test

1. A
2. D
3. A
4. E
5. C
6. D
7. C
8. B
9. C
10. B
11. B
12. B
13. A
14. C
15. C
16. E
17. A
18. A
19. C
20. A
21. C
22. D
23. A
24. A
25. C
26. B
27. E
28. D
29. D
30. C

1. $\sqrt{3-2x} \geq 0$

$3-2x \geq 0$

$-2x \geq -3$

$\boxed{x \leq \frac{3}{2}}$

2. $\lim_{x \to 3^-} \sqrt{9-x^2} = 0$

$\lim_{x \to 3^+} \sqrt{9-x^2}$ = Does not exist

$\boxed{\therefore \lim_{x \to 3} \sqrt{9-x^2} \text{ Does not exist}}$

3. $\lim_{x \to -1^-} x+3 = \lim_{x \to -1^+} 2x-c$

$\lim_{x \to -1^-} (-1+3) = \lim_{x \to -1^+} (-2-c)$

$2 = -2 - c$

$\boxed{c = -4}$

4. $y = x^4(2x-3)^5 \sin x \cos 2x$

$y = 4x^3(2x-3)^5 \sin x \cos 2x$

$+10x^4(2x-3)^4 \sin x \cos 2x$

$+ x^4(2x-3)^5 \cos x \cos 2x$

$- 2x^4(2x-3)^5 \sin x \sin 2x$

5. $\int (2x^4 + 3x^2 - 2x)\,dx$

$\boxed{= \frac{2}{5}x^5 + x^3 - x^2 + C}$

6. $f(x) = 3x^4 - 4x$

$f''(x) = x^2 - 4$

$x^2 - 4 = 0 \Rightarrow x = \pm 2$

$\boxed{f(2) = \frac{25}{3} \quad f(-2) = \frac{25}{3}}$

7. $128\pi = \pi r^2 h$ $\quad 128\pi = (1.5)^2 \pi h$

$0 = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$

$0 = 2(.05)(56.\overline{8}) + (1.5)\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} \approx 3.793}$

8. $\int_{-5}^{0} (x^3 + x^2 - 12x) - (-x^2 + 3x)\,dx +$

$\int_{0}^{3} (-x^2 + 3x) - (x^3 + x^2 - 12x)\,dx$

$\int_{-5}^{0} (x^3 + 2x^2 - 15x)\,dx + \int_{0}^{3} (-x^3 - 2x^2 + 15x)\,dx$

$\left[\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{15}{2}x^2\right]_{-5}^{0} + \left[-\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{15}{2}x^2\right]_{0}^{3}$

$\boxed{= \frac{863}{6}}$

9. $\frac{dy}{dt} = 4x \frac{dx}{dt}$

$-2 = 4\left(-\frac{3}{2}\right)\frac{dx}{dt}$

$-2 = -6\frac{dx}{dt}$

$\boxed{\frac{1}{3} = \frac{dx}{dt}}$

10. From graph.

11. $\lim_{x \to 2} \frac{x-2}{x^2-4}$

$\lim_{x \to 2} \frac{x-2}{(x-2)(x+2)}$

$\lim_{x \to 2} \frac{1}{x+2} = \boxed{\frac{1}{4}}$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$\boxed{f(1) = -1 \quad f(-1) = -3}$

13. $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$f'(x_n) = 3x_n^2 + 1$

$\boxed{x_{n+1} = x_n - \dfrac{x_n^3 + x_n + 1}{3x_n^2 + 1}}$

14. $|4x-4| = 4|x-1|$

$4|x-3| < .001$

$|x-3| < \dfrac{.001}{4}$

$\boxed{|x-3| < .00025}$

15. $f(x) = -x^3 + 9x^2 - 24x + 18$

$f'(x) = -3x^2 + 18x - 24$

$-3(x^2 - 6x + 8) = 0$

$(x-4)(x-2) = 0$

$x = 4, 2$

$(-\infty, 2)$	$(2, 4)$	$(4, \infty)$
$f'(0) < 0$	$f'(3) > 0$	$f'(5) < 0$
dec.	inc.	dec

16. $y = x^{\sin x}$

$\ln y = \sin x \ln x$

$\dfrac{y'}{y} = \cos x \ln x + \dfrac{1}{x} \sin x$

$y' = x^{\sin x}\left[\cos x \ln x + \dfrac{\sin x}{x}\right]$

$= \dfrac{2(x+4) - (x+2)}{(x+4)^2}$

$x = -2$

$(-\infty, -2)$	$(-2, \infty)$
$f'(-3) < 0$	$f'(0) > 0$

$\boxed{f(-2) = -1 \Rightarrow \text{min.}}$

18. $\int x\sqrt{3-7x^2}\,dx \qquad u = 3-7x^2$

$du = -14x\,dx$

$-\frac{1}{14}\int u^{1/2}\,du$

$-\frac{1}{14}\left(\frac{2}{3}u^{3/2}\right) + C$

$\boxed{-\frac{1}{21}(3-7x^2)^{3/2} + C}$

19. $v = -35t + 58$

at $t = 1 \Rightarrow v = -35 + 58$

$\boxed{v = 23}$

20. (A) $\lim_{x\to\infty} \dfrac{x^3}{1-2x^3}$

$\lim_{x\to\infty} \dfrac{x^3/x^3}{1/x^3 - 2x^3/x^3}$

$\lim_{x\to\infty} \dfrac{1}{1/x^3 - 2} = -\dfrac{1}{2}$

21. $= \dfrac{3-2}{2(4)}\left[f(2) + 2f(2.25) + 2f(2.5) + 2f(2.75) + f(3)\right]$

$= \frac{1}{8}\left[1 + 2(.64) + 2(.\overline{4}) + 2(.32653) + (.25)\right]$

$\boxed{\approx 0.5090}$

22. $10x - 2y - 2xy' + 14yy' = 0$

$y'(-2x + 14y) = -10x + 2y$

$\boxed{y' = -\dfrac{10x - 2y}{-2x + 14y}}$

$\frac{dT}{dx} = 1.8 - \frac{2x}{43{,}000}$

$= 1.8 - \frac{x}{21{,}500}$

24. $f(x) = (x^2 - 4x)(x^{-1/2})$

$f(x) = x^{3/2} - 4x^{1/2}$

$f'(x) = \frac{3}{2}x^{1/2} - 2x^{-1/2}$

$= \frac{3}{2}x^{1/2} - \frac{2}{x^{1/2}}$

25. $(-\infty, 6) \quad (6, \infty)$

$+ \quad -$

$\therefore f'(x)$ is + on $(-\infty, 6)$ and − on $(6, \infty)$

26. $s = \int_2^4 \sqrt{1 + [(x-2)^{1/2}]^2}\, dx$

$s = \int_2^4 \sqrt{x-1}\, dx \quad u = x-1, \; du = dx, \; x=2, u=1, \; x=4, u=3$

$= \int_1^3 u^{1/2}$

$= \frac{2}{3}u^{3/2}\Big]_1^3$

$= \frac{2}{3}\left[3^{3/2} - 1\right]$

27. $f'(x) = 3(x^{1/2} + x)^2\left(\frac{1}{2}x^{-1/2} + 1\right)$

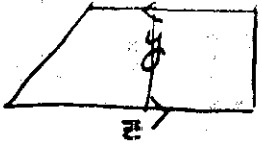
$f'(4) = 3(2+4)^2\left(\frac{1}{2 \cdot 2} + 1\right)$

$f'(4) = 135$

28. $f'(x) = \cos x$

$f'\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$



$\frac{z-x}{30} = \frac{y-x}{20}$

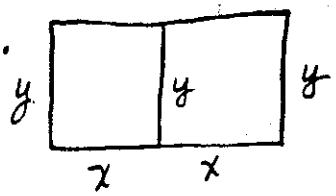
$2z = 3y - x$

$2\frac{dz}{dt} = 3\frac{dy}{dt} - \frac{dx}{dt}$

$2(-30) = 3(25) - \frac{dx}{dt}$

$\frac{dx}{dt} = 135$

30.



$P = 4x + 3y$

$160 = 4x + 3y$

$y = \frac{160 - 4x}{3}$

$A = 2xy$

$A = 2x\left(\frac{160 - 4x}{3}\right)$

$A' = \frac{1}{3}(320 - 16x)$

$320 - 16x = 0$

$-16x = -320$

$x = 20$

$y = \frac{80}{3}$