1999 Mu Alpha Theta Tennessee Bowl
Theta Division

Individual Questions:

1. The largest number of consecutive zero place holders that would appear in the product of 11625000 and 102120000 is what?

Answer: 9
Solution: checking out the products 12 (25) = 300 and 625 (12) = 7500 which implies that you will get an additional 2 zeros in addition to the 3 zeros from the first number and 4 numbers from the second number therefore 9 zeros total.

2. Three vertices of a regular hexagon are chosen at random. What is the probability that an isosceles triangle will be formed by connecting these three vertices.

Answer: $\frac{2}{5}$

Solution: ${}_6C_3 = 20$ total possibilities. There are six isosceles triangles and 2 equilateral triangles

3. If $\operatorname{Log}_2 X + \operatorname{Log}_4 X - \operatorname{Log}_8 X = 7$, then X equals what?

Answer: 64

Solution: $\operatorname{Log}_2 X + \operatorname{Log}_4 X - \operatorname{Log}_8 X = 7$;

$Log_2 X + Log_{2^2} X - Log_{2^3} X = 7; Log_2 X + \frac{1}{2} Log_2 X - \frac{1}{3} Log_2 X = 7; \frac{7}{6} Log_2 X = 7;$ $\operatorname{Log}_2 X = 6$; $x = 64$

4. If $P(x + 2) = x^3 + 9x^2 + 26x + 23$, then $P(x - 2)$ is what?

Answer: $X^3 - 3X^2 + 2X - 1$

Solution: $P(x - 2) = P(x - 4) = (x - 4)^3 + 9(x - 4)^2 + 26(x - 4) + 23 =$
$x^3 - 12x^2 + 48x - 64 + 9x^2 - 72x + 144 + 26x - 104 + 23 = X^3 - 3X^2 + 2X - 1$

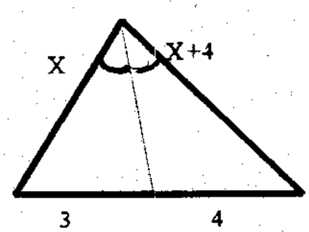
1999 Mu Alpha Theta Tennessee Bowl
Theta Division

Group Questions:

5. One side of a triangle is 4 inches longer than another side. The ray bisecting the angle between these two sides divides the third side into 3 inch and 4 inch segment. Find the perimeter, in inches, of the triangle.

Answer: 35

Solution: $\frac{X}{3} = \frac{X+4}{4}$; $4X = 3X + 12$; $X = 12$; $P = 12 + 16 + 3 + 4$



6. A RUNSUM is a sequence of consecutive positive integers. Three is the smallest RUNSUM of two consecutive integers. (1 + 2). Nine is the first number that has two different RUNSUMS: (2 + 3 + 4 and 4 + 5). Fifteen is the first number that has three RUNSUMS: (1 + 2 + 3 + 4 + 5 and 7 + 8 and 4 + 5 + 6) What is the smallest number with four different RUNSUMS?

Answer: 45
Solution: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 and 7 + 8 + 9 + 10 + 11 and 14 + 15 + 16 and 22 + 23

7. Find the area of the triangle formed using the centers of the following conic sections.
$X^2 + Y^2 - 4X - 10Y + 25 = 0$; $3X^2 + 4Y^2 - 24X + 8Y + 40 = 0$; $3X^2 - 2Y^2 + 36X - 8Y + 94 = 0$

Answer: 23

Solution: In graphing form the three conic equations are: $(x - 2)^2 + (y - 5)^2 = 4$; $\frac{(x-4)^2}{4} + \frac{(y+1)^2}{3} = 1$; $\frac{(x+6)^2}{2} - \frac{(y+2)^2}{3} = 1$ The centers are (2, 5), (4, - 1), (- 6, - 2). Using matrices you can find the area to be 23. $\pm\frac{1}{2}\begin{vmatrix} 4 & 1 & 1 \\ -6 & -2 & 1 \\ 2 & 5 & 1 \end{vmatrix} = 23$

8. For a given line, the sum of the x and y-intercepts is $6\frac{1}{3}$ and the product is 10. Find all possible value for the slope of this line.

Answer: $-\frac{9}{10}, -\frac{10}{9}$

Solution: Intercepts: (x,0), (0,y) Slope = $\frac{0-y}{x-0} = \frac{-y}{x}$ $xy = 10 \Rightarrow y = \frac{10}{x}$ $x + y = x + \frac{10}{x} = \frac{19}{3}$

$3x^2 - 19x + 30 = 0$ $(3x - 10)(x - 3) = 0$ $x = 3 \Rightarrow y = \frac{10}{3}$ Slope = $\frac{-10}{9}$ $x = \frac{10}{3}$ $y = 3$ slope = $\frac{-9}{10}$

9A. Find all rational zeros of $f(x) = 4x^3 - 12x^2 - 19x + 12$

Answer: $4, \frac{1}{2}, -\frac{3}{2}$

Solution: $4x^3 - 12x^2 - 19x + 12 = 0$; $(x - 4)(2x-1)(2x+3) = 0$; $x = 4, \frac{1}{2}, -\frac{3}{2}$

9B. Find the area bounded by the functions $f(x) = -\frac{2}{3}(x+4)^2 + 3$ and $f(x) = -3$

Answer: 24 units ^{2}

Solution: Area of a parabolic section of $A = \frac{2}{3}bh$ The vertex is (- 4, 3) and when y = - 3 you get (- 7, - 3) and (- 1, - 3). The function $f(x) = -3$ creates a base of length 6 and a height of 6. $A = \frac{2}{3}(6)(6) = 24$

10A. Solve for x over the Real Numbers: $8^{\frac{1}{6}} + x^{\frac{1}{3}} = \frac{7}{3 - \sqrt{2}}$

Answer: 27

Solution: $8^{\frac{1}{6}} + x^{\frac{1}{3}} = \frac{7}{3 - \sqrt{2}}$ $(2^3)^{\frac{1}{6}} + x^{\frac{1}{3}} = 3 + \sqrt{2}$ $x^{\frac{1}{3}} = 3$ $x = 27$

10B. Find the positive value of m such that P (m, 2m) is 5 units from the line $12x + 5y = 1$.

Answer: 3

Solution: $\frac{|12m + 5(2m) - 1|}{\sqrt{144 + 25}} = 5$ $|22m - 1| = 65$ $22m - 1 = 65$ or $22m - 1 = -65$; $m = 3$, $m = \frac{-32}{11}$

11A. Find $g(4)$ if $g(x-1) = x^2 + 2$

Answer: 27
Solution: If $x = 1$ then $g(0) = 2$; if $x = 2$ then $g(1) = 6$; if $x = 3$ then $g(2) = 11$; if $x = 4$ then $g(3) = 18$; if $x = 5$ then $g(4) = 27$

11B. Find $g^{-1}(4)$ if $g(x) = x^3 - 4$

Answer: 2
Solution: $y = g(x) = x^3 - 4$ $\quad x = y^3 - 4$ $\quad y = \sqrt[3]{x+4}$ $\quad g^{-1}(4) = \sqrt[3]{4+4} = \sqrt[3]{8} = 2$

11C. Find $f^{-1}(g(h(5)))$ if $f(x) = x - 1$, $g(x) = 3x$, and $h(x) = \dfrac{5}{x}$

Answer: 4
Solution: $h(5) = 1$; $g(1) = 3$; $f^{-1}(3) = 4$

11D. Find the value of A if the point (3, 2) lies on the graph of the inverse of $f(x) = 2x^2 + x + A$

Answer: - 15
Solution: If (3, 2) lies on the inverse of f, then the point (2, 3) lies on f. Substituting in f gives $2 \bullet 2^3 + 2 + A = 3$; $A = -15$

12A. Given $f = \{(0,-3), (2,5), (-1,1), (4,2)\}$, $g = \{(-1,2), (1,4), (4,3), (0,-1)\}$, and $h = \{(4,2), (1,0), (-3,4), (3,-1)\}$, find $f(g(h(x)))$.

Answer: $\{(1,1), (3,5)\}$
Solution: Starting with set h the only ordered pairs that are able to have a value for $f(g(h(x)))$ are
$(1,0): 1 \to 0 \to -1 \to 1$; $\quad (3,5): 3 \to -1 \to 2 \to 5$

12B. Solve for all values of x: $2^x + 2^x + 2^x + 2^x = 2^{x^2}$

Answer: 2, -1

Solution: $2^x + 2^x + 2^x + 2^x = 2^{x^2}$; $4(2^x) = 2^{x^2}$ $2^{x+2} = 2^{x^2}$; $x^2 = x + 2$; $x^2 - x - 2 = 0$
$(x-2)(x+1)=0$; $x = 2, x = -1$

12C. In the expansion of $(2x + c)^8$, for some non-zero value of the constant c, the fourth and fifth terms have the same numerical coefficient. Find the value of c.

Answer: $\frac{8}{5}$

Solution: ${}_8C_3 \bullet 2^5 \bullet c^3 = {}_8C_4 \bullet 2^4 \bullet c^4$ $56 \bullet 2 = 70 \bullet c$ $c = \frac{8}{5}$

12D. Find the smallest positive integer which has exactly 27 distinct positive integral factors.

Answer: 900

Solution: A number of the form $(a^2)(b^2)(c^2)$ with a, b, and c prime has $(2+1)(2+1)(2+1) = 27$ factors
The smallest such integer is $2^2 \bullet 3^2 \bullet 5^2 = 900$