

# 1992 NATIONAL MU ALPHA THETA CONVENTION

## THETA STATE BOWL

### THETA STATE #1

Find all values of  $k$  for which the positive difference between the solutions of  $5x^2 + 4x + k = 0$  equals the sum of the squares of these solutions.

### THETA STATE #2

Find the ordered pairs of positive integers  $(a, b)$ , with  $a < b$ , for which

$$\sqrt{1 + \sqrt{21 + 12\sqrt{3}}} = \sqrt{a} + \sqrt{b}.$$

### THETA STATE #3

Let  $[x]$  denote the greatest integer function, and  $\{x\} = x - [x]$  is called the fractional part of  $x$ . If

$$z = \frac{\{\sqrt{3}\}^2 - 2\{\sqrt{2}\}^2}{\{\sqrt{3}\} - 2\{\sqrt{2}\}}, \text{ find } [z].$$

### THETA STATE #4

Solve for  $x$ :  $\frac{|x-2|}{x} > x$

### THETA STATE #5

If  $f(1) = 1$ ,  $f(2) = 2$ ,  $f(n) = f(n-1) + f(n-2)$  for all natural numbers  $n > 2$ , find  $f(12)$ .

THETA STATE #6

A sphere of volume  $288\pi$  cubic inches is to be coated uniformly with lead. How thick must the lead be so that the surface of the exposed lead is three times that of the original sphere? Leave your answer in simplest radical form.

THETA STATE #7

A train,  $x$  meters long, traveling at a constant speed, takes 20 seconds from the time it first enters a tunnel 300 meters long until the time it completely emerges from the tunnel. One of the stationary ceiling lights in the tunnel is directly above the train for 10 seconds. Find  $x$ .

THETA STATE #8

With seamstress X making  $x$  dresses per day and seamstress Y making  $y$  dresses per day, X requires 5 more days than does Y to make 60 dresses. If each seamstress were to increase her daily output by 1 dress, X would need only 3 days more than Y to make 60 dresses. Find  $(x,y)$ .

THETA STATE #9

The three positive integers 3, 36,  $x$  are such that their arithmetic mean exceeds their geometric mean by 13. Find  $x$ .

THETA STATE #10

Two nonintersecting circles have radii 1 and 9, and one of their common external tangents is 12 units long. A third circle is tangent to the first two and also to one of their common external tangents. Find the radius of the third circle.

THETA STATE #11

A freighter sailing due north at 12 mph sights a cruiser straight ahead at an unknown distance and speeding due east at unknown speed. After 15 minutes, the vessels are 10 miles apart and 15 minutes later, they are 13 miles apart. How far apart are the vessels when the cruiser is due east of the freighter?

THETA STATE #12

Let  $f$  be a function defined by  $f(x) =$

$$1 + \frac{1}{2 - \frac{1}{3 + \frac{1}{4 - \frac{1}{x}}}}$$

Find all zeros of  $f$ .

THETA STATE #13

If  $f$  is a function of direct variation, and if  $f^{-1}(x) = \frac{4}{9} f(x)$ , determine  $f(\frac{9}{4})$ .