

MU ALPHA THETA - HAWAII 1993

Topic Test

Simplifying Fractional Expressions - Theta Division

ANSWER KEY

1. D

2. D

3. E

4. A

5. C

6. B

7. B

8. A

9. E

10. D

11. D

12. C

13. D

14. A

15. B

16. E

17. A

18. C

19. D

20. C

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Simplifying Fractional Expressions Topic Test - Theta Division
(for students who have NOT had more math than Algebra I, Algebra II, and Geometry)

NO CALCULATORS

FOR ALL PROBLEMS ON THIS TEST, ASSUME NON-ZERO DENOMINATORS

1. Simplify:

$$\frac{2}{x} + \frac{3}{xy}$$

- A) $2y+3$ B) $\frac{5}{x+xy}$ C) $\frac{2x+3y}{x^2y}$ **D) $\frac{2y+3}{xy}$** E) none of these

$$= \frac{2y}{xy} + \frac{3}{xy} = \frac{2y+3}{xy}$$

2. Simplify:

$$1^{-1} + 2^{-1} + 3^{-1}$$

- A) 3^{-3} B) 6^{-3} C) -6 **D) $\frac{11}{6}$** E) none of these

$$= 1 + \frac{1}{2} + \frac{1}{3} = \frac{6}{6} + \frac{3}{6} + \frac{2}{6} = \frac{11}{6}$$

3. Simplify:

$$\frac{2\frac{1}{4}}{3\frac{1}{12}}$$

- A) $-1\frac{1}{6}$ B) $\frac{9}{37}$ C) $\frac{1}{3}$ D) $\frac{13}{49}$ **E) none of these**

$$= \frac{\frac{9}{4}}{\frac{37}{12}} = \frac{9}{4} \cdot \frac{12}{37} = \frac{27}{37}$$

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4. Simplify:

$$\frac{\frac{x+y}{x-y} - \frac{x-y}{x+y}}{1 + \frac{y^2}{x^2 - y^2}}$$

- A) $\frac{4y}{x}$ B) $\frac{4y}{x^2}$ C) $\frac{-4y}{x}$ D) $\frac{-4y}{x^2}$ E) none of these

$$\begin{aligned} &= \frac{\frac{(x+y)^2 - (x-y)^2}{(x-y)(x+y)}}{\frac{x^2 - y^2 + y^2}{x^2 - y^2}} = \frac{[(x+y) + (x-y)][(x+y) - (x-y)]}{x^2} \\ &= \frac{[2x][2y]}{x^2} = \frac{4y}{x} \end{aligned}$$

5. Evaluate:

$$1 + \frac{2}{1 + \frac{2}{1+4}}$$

- A) $\frac{10}{7}$ B) $\frac{19}{5}$ C) $\frac{17}{7}$ D) $\frac{11}{6}$ E) $\frac{9}{5}$

$$= 1 + \frac{2}{1 + \frac{2}{5}} = 1 + \frac{2}{\frac{7}{5}} = 1 + \frac{10}{7} = \frac{17}{7}$$

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6. Simplify:

$$\frac{a^{-2} - b^{-2}}{a^{-1}b^{-1}}$$

- A) $\frac{a^2 - b^2}{ab}$ B) $\frac{b^2 - a^2}{ab}$ C) $\frac{a^2 - b^2}{a^3b^3}$ D) $\frac{b^2 - a^2}{a^3b^3}$ E) none of these

$$= \frac{\frac{1}{a^2} - \frac{1}{b^2}}{\frac{1}{a} \cdot \frac{1}{b}} = \frac{\frac{b^2 - a^2}{a^2b^2}}{\frac{1}{ab}} = \frac{b^2 - a^2}{a^2b^2} \cdot ab = \frac{b^2 - a^2}{ab}$$

7. Simplify:

$$\frac{2x}{1-2x} + \frac{3x}{2x+1} - \frac{3}{4x^2-1}$$

- A) $\frac{x-3}{1-2x}$ B) $\frac{3-x}{1-2x}$ C) $\frac{2x^2-5x+3}{4x^2-1}$ D) $\frac{2x^2+5x-3}{4x^2-1}$ E) $\frac{x+3}{1-2x}$

$$= \frac{2x(1+2x) + 3x(1-2x) + 3}{1-4x^2}$$

$$= \frac{2x + 4x^2 + 3x - 6x^2 + 3}{1-4x^2}$$

$$= \frac{-2x^2 + 5x + 3}{1-4x^2} = \frac{(1+2x)(3-x)}{(1+2x)(1-2x)} = \frac{3-x}{1-2x}$$

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8. Simplify:

$$\frac{x}{1 - \frac{1}{\frac{y+x}{y}}}$$

- A) $x+y$ B) $\frac{1}{x+y}$ C) $\frac{x^2}{x+y}$ D) $\frac{x+y}{x^2}$ E) none of these

$$= \frac{x}{1 - \frac{y}{x+y}} = \frac{x}{\frac{x+y-y}{x+y}} = x \cdot \left(\frac{x+y}{x}\right) = x+y$$

9. Simplify:

$$\frac{x^2 + \frac{1}{x}}{x^2 - \frac{1 - \frac{1}{x^2}}{\frac{1}{x} + \frac{1}{x^2}}}$$

- A) $\frac{x}{x+1}$ B) $\frac{1}{x}$ C) $\frac{1}{x^2+x}$ D) x^2+x E) $\frac{x+1}{x}$

$$= \frac{\frac{x^3+1}{x}}{x^2 - \frac{\frac{x^2-1}{x^2}}{\frac{x+1}{x^2}}} = \frac{\frac{x^3+1}{x}}{x^2 - \frac{x^2-1}{x+1}} = \left(\frac{x^3+1}{x}\right) \left(\frac{x+1}{x^3+x^2-x^2+1}\right) = \frac{x+1}{x}$$

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10. $\left(\frac{3}{x^2+2x-8} - \frac{1}{x^2-4} - \frac{1}{x^2+6x+8}\right)\left(\frac{1}{2}x+1\right) =$

A) $\frac{1}{2(x-2)(x-4)}$ B) $\frac{1}{x-2}$ C) $\frac{1}{x+2}$ D) $\frac{1}{2x-4}$ E) $\frac{1}{2x+4}$

$$= \left(\frac{3}{(x+4)(x-2)} - \frac{1}{(x-2)(x+2)} - \frac{1}{(x+4)(x+2)}\right)\left(\frac{x+2}{2}\right)$$

$$= \left(\frac{(3x+6) - (x+4) - (x-2)}{(x+4)(x+2)(x-2)}\right)\left(\frac{x+2}{2}\right)$$

$$= \frac{\cancel{(x+4)}\cancel{(x+2)}}{2\cancel{(x+2)}\cancel{(x+2)}(x-2)} = \frac{1}{2x-4}$$

11. Simplify:

$$\frac{x - \frac{x-1}{x+1}}{x-1} - \frac{x - \frac{x+1}{x-1}}{x+1}$$

A) $\frac{1}{x-1}$ B) $\frac{x+2}{x-1}$ C) 0 D) $\frac{2}{x-1}$ E) $\frac{1}{x+1}$

$$= \frac{x(x+1) - (x-1)}{(x+1)(x-1)} - \frac{x(x-1) - (x+1)}{(x+1)(x-1)}$$

$$= \frac{(x^2 + x - x + 1) - (x^2 - x - x - 1)}{(x+1)(x-1)}$$

$$= \frac{2x + 2}{(x+1)(x-1)}$$

$$= \frac{2}{x-1}$$

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12. $\frac{1}{2x} - \frac{1}{3x} + \frac{1}{4x} =$

- A) $\frac{1}{8x}$ B) $\frac{13}{12x}$ C) $\frac{5}{12x}$ D) $\frac{5}{24x}$ E) $\frac{1}{3x}$

$$= \frac{6 - 4 + 3}{12x}$$

$$= \frac{5}{12x}$$

13. Simplify the following:

$$\frac{2}{x^2 - 3x + 2} + \frac{2}{x^2 - x - 2} + \frac{1}{1 - x^2}$$

- A) $\frac{3x-2}{(x-1)(x+1)(x-2)}$ B) $\frac{5x-2}{(x-1)(x+1)(x-2)}$ C) $\frac{5x+2}{(x-1)(x+1)(x-2)}$
 D) $\frac{3x+2}{(x-1)(x+1)(x-2)}$ E) $\frac{5x+6}{(x-1)(x+1)(x-2)}$

$$= \frac{2}{(x-1)(x-2)} + \frac{2}{(x-2)(x+1)} + \frac{-1}{(x-1)(x+1)}$$

$$= \frac{2(x+1) + 2(x-1) - 1(x-2)}{(x-1)(x+1)(x-2)}$$

$$= \frac{2x + 2 + 2x - 2 - x + 2}{(x-1)(x+1)(x-2)}$$

$$= \frac{3x+2}{(x-1)(x+1)(x-2)}$$

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14. $\frac{ac + bc + ad + bd}{ac - ad - bc + bd} + \frac{a^2 + 2ab + b^2}{c^2 - 2cd + d^2} \times \frac{a-b}{c+d} =$

- A) $\frac{c-d}{a+b}$ B) $\frac{(c-d)(c+d)}{(a+b)(a-b)}$ C) $\frac{c-d}{a-b}$ D) $\frac{c+d}{a-b}$ E) $\frac{(c-d)^2}{(a+b)^2}$

$$= \frac{c(a+b) + d(a+b)}{c(a-b) - d(a-b)} \cdot \frac{(c-d)(c-d)}{(a+b)(a+b)} \cdot \frac{(a-b)}{(c+d)}$$

$$= \frac{\cancel{(c+d)}(a+b)(c-d)\cancel{(c-d)}\cancel{(a-b)}}{\cancel{(c-d)}\cancel{(a-b)}(a+b)(a+b)\cancel{(c+d)}}$$

$$= \frac{c-d}{a+b}$$

15. Simplify:

$$\frac{x^2 - 9}{x^3 + x^2 - 9x - 9}$$

- A) $\frac{1}{x-1}$ B) $\frac{1}{x+1}$ C) $\frac{1}{x+3}$ D) $\frac{1}{x-3}$ E) none of these

$$= \frac{(x+3)(x-3)}{x^2(x+1) - 9(x+1)}$$

$$= \frac{(x+3)(x-3)}{(x^2-9)(x+1)}$$

$$= \frac{1}{x+1}$$

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16. $a + (a + a^{-1})^{-1} =$

- A) $\frac{2a^2+1}{a}$ B) $\frac{a^2+2a}{a^2+1}$ C) a D) $\frac{a^2+2}{a(a^2+1)}$ **E) none of these**

$$= a + \frac{1}{a + a^{-1}} = a + \frac{1}{a + \frac{1}{a}} = a + \frac{1}{\frac{a^2+1}{a}} = a + \frac{a}{a^2+1}$$

$$= \frac{a^3+a+a}{a^2+1} = \frac{a^3+2a}{a^2+1} = \frac{a(a^2+2)}{a^2+1}$$

17. Simplify:

$$\frac{a^4 - 10a^2 + 169}{a^2 + 6a + 13}$$

- A) $a^2 - 6a + 13$** B) $a^2 + 6a + 13$ C) $a^2 - 5a + 13$ D) $a^2 + 5a + 13$ E) none of these

$$= \frac{(a^4 + 26a^2 + 169) - 36a^2}{a^2 + 6a + 13}$$

$$= \frac{(a^2 + 13)^2 - (6a)^2}{a^2 + 6a + 13}$$

$$= \frac{(a^2 - 6a + 13)(a^2 + 6a + 13)}{(a^2 + 6a + 13)}$$

$$= a^2 - 6a + 13$$

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18. Simplify:

$$\frac{y^{-1} + 1}{y - 1}$$

- A) $\frac{1}{y(y-1)}$ B) $\frac{y(y+1)}{y-1}$ C) $\frac{y+1}{y(y-1)}$ D) $\frac{y+1}{y-1}$ E) none of these

$$= \frac{\frac{1}{y} + 1}{y - 1} = \frac{y + 1}{y} \cdot \frac{1}{y - 1} = \frac{y + 1}{y(y - 1)}$$

19. Simplify:

$$\frac{b^4 - 16}{b^4 - 4b^3 + 8b^2 - 16b + 16}$$

- A) $\frac{b^2 - 4}{b^2 + 4}$ B) $\frac{b - 2}{b + 2}$ C) $\frac{b^2 + 4}{1 - 4b}$ D) $\frac{b + 2}{b - 2}$ E) none of these

$$\begin{aligned} &= \frac{b^4 - 16}{(b^4 + 8b^2 + 16) - (4b^3 - 16b)} \\ &= \frac{(b^2 + 4)(b^2 - 4)}{(b^2 + 4)^2 - 4b(b^2 + 4)} \\ &= \frac{\cancel{(b^2 + 4)}(b^2 - 4)}{\cancel{(b^2 + 4)}(b^2 - 4b + 4)} \\ &= \frac{b^2 - 4}{(b - 2)^2} \\ &= \frac{\cancel{(b - 2)}(b + 2)}{\cancel{(b - 2)}(b - 2)} = \frac{b + 2}{b - 2} \end{aligned}$$

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20. Simplify:

$$\frac{m^{-1} + n^{-1}}{m^{-3} + n^{-3}}$$

- A) $\frac{m^2n^2}{(m-n)^2}$ B) $\frac{m^2n^2}{m^2+mn+n^2}$ C) $\frac{m^2n^2}{m^2-mn+n^2}$ D) $\frac{m^2n^2}{(m+n)^2}$ E) none of these

$$= \frac{\frac{1}{m} + \frac{1}{n}}{\frac{1}{m^3} + \frac{1}{n^3}}$$

$$= \frac{m+n}{mn} \cdot \frac{m^3n^3}{m^3+n^3}$$

$$= \frac{(m+n)(m^3n^3)}{mn(m+n)(m^2-mn+n^2)}$$

$$= \frac{m^2n^2}{m^2-mn+n^2}$$