

Sequences and Series Topic Test

1) Find the sum of the series $1 - 1/3 + 1/9 - 1/27 + \dots$

- A) $1/2$ B) $4/5$ C) $2/3$ D) $3/4$ E) N.O.T.A.

2) Find the 22nd digit after the decimal point in the decimal expansion of $1/7$.

- A) 1 B) 4 C) 5 D) 7 E) N.O.T.A.

3) Find the sum of the first thirty terms of the series:

$$1 - 2 + 2 - 4 + 3 - 6 + 4 - 8 \dots$$

- A) -120 B) -105 C) -465 D) -15 E) N.O.T.A.

4) The second term of a geometric sequence is 4 and the sixth term is 16. Find the fourth term if the ratio of consecutive terms is a real number.

- A) 10 B) 8 C) $8\sqrt{2}$ D) 12 E) N.O.T.A.

5) Terms a, b, c, d are the first four terms of an arithmetic sequence with $a < b < c < d$. If $d - a = r$, what is $c - a$?

- A) $r/3$ B) $r/2$ C) $2r/3$ D) $3r/4$ E) N.O.T.A.

6) Evaluate $(-i)^{15}$ where $i = \sqrt{-1}$.

- A) 1 B) -1 C) -i D) i E) N.O.T.A.

7) Find the next term in the sequence :

$$50, 40, 32, 26, 22, 20, \dots$$

- A) 22 B) 20 C) 18 D) 16 E) N.O.T.A.

8) Find the fourth term of an arithmetic sequence whose first term is 1 and seventh term is 17.

- A) $7 \frac{1}{3}$ B) $8 \frac{2}{3}$ C) 9 D) 8 E) N.O.T.A.

9) A man starts a trip travelling 5 mi/hr. At the end of each hour, he increases his speed 5 mi/hr. How long does it take him to travel 250 mi?

- A) 8 hr B) $7 \frac{2}{3}$ hr C) $9 \frac{1}{2}$ hr D) $9 \frac{5}{9}$ hr E) N.O.T.A.

10) Evaluate $\prod_{i=1}^{20} \left(\frac{i}{i+1}\right)$.

- A) $\frac{1}{20}$ B) $\frac{1}{21}$ C) $\frac{2}{19}$ D) $\frac{1}{10}$ E) N.O.T.A.

11) Which of the following is equal to $\frac{1}{1+2x^2}$ for $\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2}$?

- A) $1 - 2x^2 + 4x^4 - 8x^8 + \dots$
 B) $1 + 2x^2 + 4x^4 + 8x^8 + \dots$
 C) $1 + 2x^2 + 4x^4 + 6x^6 + \dots$
 D) $1 - 2x^2 + 4x^4 - 6x^6 + \dots$
 E) N.O.T.A.

12) $\sum_{n=1}^{\infty} \frac{n}{(n+1)(n+2)} =$

- A) 2 B) 1 C) $\frac{3}{2}$ D) 4 E) N.O.T.A.

13) Find x if $x = \frac{2}{2 + \frac{4}{2 + \frac{4}{2 + \frac{4}{2 + \dots}}}}$

- A) $-1 + \sqrt{3}$ B) $\frac{-1 + \sqrt{5}}{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\sqrt{5} - \sqrt{2}$ E) NOT

14) Let a and b be the fifth and sixth terms, respectively, of the sequence: $\frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{1}{4}, a, b$. Find a/b.

- A) $\frac{6}{5}$ B) $\frac{5}{3}$ C) $\frac{8}{5}$ D) $\frac{3}{2}$ E) N.O.T.A.

15) If the terms $\ln(a)$, $\ln(b)$, $\ln(c)$, $\ln(d)$ form an arithmetic sequence with common difference 1, then the terms a , b , c , d are :

- A) an arithmetic sequence with difference e
- B) a geometric sequence with ratio e
- C) an arithmetic sequence with ratio one
- D) a geometric sequence with ratio $1/e$
- E) N.O.T.A.

16) The expression $\frac{\sum_{i=0}^n \binom{n}{i}}{\sum_{i=0}^m \binom{m}{i}}$ is an integer if and only if :

- A) $n=m$
- B) $n>m$
- C) $n \geq m$
- D) n is a multiple of m
- E) N.O.T.A.

17) Evaluate $\sum_{i=1}^{10} \sum_{k=1}^i \binom{i}{k}$.

- A) 4094
- B) 2047
- C) 2036
- D) 4095
- E) N.O.T.A.

18) What is the remainder when 2^{71} is divided by 9?

- A) 7
- B) 8
- C) 2
- D) 4
- E) N.O.T.A.

19) Evaluate the sum : $1(1) + 3(2) + 5(3) + 7(4) + \dots + 49(25)$.

- A) 10375
- B) 9375
- C) 10725
- D) 5200
- E) N.O.T.A.

20) Which of the following are true : $\sum_{i=0}^{\infty} a_i$ diverges to infinity if

- i) $a_m \geq x$ for some $x > 0$ for infinitely many values of m
- ii) $a_m > 0$ for infinitely many values of m
- iii) $a_m < x$ for some $x < 0$ for infinitely many values of m

- A) i and ii only
- B) i only
- C) ii only
- D) i, ii and iii
- E) N.O.T.A.

21) Three numbers a, b, c are in an arithmetic sequence with common difference $r \geq 0$. Given that $abc = a + b + c$ and that the middle term in the sequence is a positive integer, find the sum of the squares of the five smallest possible positive values of r .

- A) 55
- B) 120
- C) 42
- D) 75
- E) N.O.T.A.

22) Find the sum of the first thirty triangular numbers, where a triangular number is a number which is the sum of the first n integers for some positive value of n .

- A) 4495
- B) 4635
- C) 4960
- D) 6975
- E) N.O.T.A.

23) Given $f(0) = 3$, $f(1) = -1$, and $f(n) = f(n-2) - f(n-1)$, find $f(100)$ in terms of the Fibonacci numbers (F_0, F_1, \dots, F_{100}), where $F_n = F_{n-1} + F_{n-2}$ and $F_0 = F_1 = 1$.

- A) $F_{99} + F_{101}$
- B) $F_{97} + 3F_{99}$
- C) $2F_{98} + F_{99}$
- D) $2F_{98} + F_{100}$
- E) N.O.T.A.

24) Trapezoid $A_1A_2A_3A_4$ is drawn with A_1A_2 parallel to A_3A_4 and $A_1A_2=8$, $A_2A_3=5$, $A_3A_4=4$, and $A_4A_1=6$. An infinite number of trapezoids are then constructed and for each one, $A_{2n-1}A_{2n}A_{2n+1}A_{2n+2}$ is similar to $A_{2n+1}A_{2n+2}A_{2n+3}A_{2n+4}$. Find the total area of all of the trapezoids.

- A) 32
- B) 24
- C) $16\sqrt{5}$
- D) $15\sqrt{7}$
- E) N.O.T.A.

25) If a harmonic series is also an arithmetic series, then the series must:

- A) diverge B) converge C) have initial term zero
D) have initial term one E) N.O.T.A.

26) Evaluate : $44 \binom{45}{0} + 43 \binom{45}{1} + 42 \binom{45}{2} + \dots + 0 \binom{45}{44} - \binom{45}{45}$.

- A) $11(2^{46})$ B) $45(2^{44})$ C) $23(2^{45})$ D) $43(2^{44})$ E) N.O.T.A.

27) Find $\sum_{n=0}^{\infty} \frac{\sin(nx)}{3^n}$ if $\sin(x) = 1/3$ and $0 \leq x \leq \pi/2$.

- A) $\frac{5+2\sqrt{2}}{34}$ B) -9 C) $\frac{2+\sqrt{2}}{15}$ D) $\frac{9+4\sqrt{2}}{49}$ E) N.O.T.A.

28) A man lists the integers from 1 to n , inclusive. He omits one of the numbers. The average of the remaining numbers is 18.8. What number did he erase?

- A) 5 B) 11 C) 8 D) 15 E) N.O.T.A.

29) Find the sum $\binom{22}{10} \binom{15}{0} + \binom{21}{9} \binom{16}{1} + \binom{20}{8} \binom{17}{2} + \dots + \binom{12}{0} \binom{25}{10}$.

- A) $\binom{38}{9}$ B) $\binom{37}{10}$ C) $\binom{37}{11}$ D) $\binom{38}{10}$ E) N.O.T.A.

30) Find the sum of the last three digits of 19^{92} .

- A) 14 B) 12 C) 16 D) 17 E) N.O.T.A.