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Binomial Theorem, Sequences + Series
Topic Test Solutions

① $a_{12} = 4(12) - 11 = 37$ A

② $a_n = a_1 + (n-1)d$ B

$94 = 3 + 26d$

$d = 3.5$

③ $768 = (6)(2)^{n-1}$ A

$128 = 2^{n-1}$

$2^7 = 2^{n-1}$

$n = 8$

④ $a_3 = 2a_2 - 3a_1 = -8$ C

$a_4 = 2a_3 - 3a_2 = -13$

$a_5 = 2a_4 - 3a_3 = -2$

⑤ 6, 12, 20, 30, 42, 56, 72, 90, 110 C

⑥ $966 = \dfrac{n[a + a + (n-1)4]}{2}$ B

$966 = \dfrac{n[12 + 4n - 4]}{2}$

$483 = n^2 + 2n$

$n^2 + 2n - 483 = 0$

$(n-21)(n+23) = 0$

$n = 21 \quad n \neq -23$

⑦ $2^{10} = 1024$ D

⑧ $\sum_{k=1}^{10} k^2 + \sum_{k=1}^{10} k + \sum_{k=1}^{10} 1$ C

$\dfrac{n(n+1)(2n+1)}{6} + \dfrac{n(1+n)}{2} + 10$

$n = 10$

$385 + 55 + 10 = 450$

(9) $4.1\overline{36}$

$10x = 41.\overline{36}$

$1000x = 4136.\overline{36}$

$990x = 4095$

$x = \frac{4095}{990} = \frac{91}{22}$

C

(10) By definition - series for e — B

(11) $S_n = \frac{a}{1-r} = \frac{40}{1-\frac{95}{100}} = 800$ — A

(12) $1{,}000{,}000 = 2\cdot 2^{n-1}$

$1000000 = 2^n$

$\log 1000000 = n \log 2$

$6 = .30n$

$n = 20$

C

(13) $\binom{14}{12}(2)^2(1)^{12}$ — B

$\frac{14\cdot 13}{1\cdot 2}\cdot 4 = 364$

(14) Geometric series with $r = \frac{52}{50}$ — E

$r > 1 \therefore$ no sum

(15) 1, 3, 6, 10, 15 21, 28, 36, 45 — E

2 out of 3 divisible by 3.

$3\overline{)1991}$ 663 with r = 2

$663 \times 2 + 1 = 1327$

(16) $\binom{16}{4}(xy)^{12}\left(-\frac{2}{y^3}\right)^4$ — C

$\frac{16\cdot 15\cdot 14\cdot 13}{1\cdot 2\cdot 3\cdot 4}\cdot 16 = 29120$

(17) It is a geometric progression.
$\therefore |r|<1$ for a sum to exist. D
Then $|b|<1$

(18) By inspection E. E

(19) $2^{\frac{1}{3}} \cdot 2^{\frac{2}{9}} \cdot 2^{\frac{3}{27}} \cdot 2^{\frac{4}{81}} \cdots$
When multiplying add exponents
$\frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \ldots$ C
Separate into different series
$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$$
$$\frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots = \frac{\frac{1}{9}}{1-\frac{1}{3}} = \frac{1}{6}$$
$$\frac{1}{27} + \frac{1}{81} + \cdots = \frac{\frac{1}{27}}{1-\frac{1}{3}} = \frac{1}{18}$$
$\frac{1}{2} + \frac{1}{6} + \frac{1}{8} + \ldots$
$$S = \frac{\frac{1}{2}}{1-\frac{1}{3}} = \frac{3}{4} \quad \therefore \text{Ans. } 2^{\frac{3}{4}}$$

(20) Let x = time for trains to meet D
$15x + 15x = 2$ distance $= rt$
$x = \frac{1}{15}$ hr $= 30 \cdot \frac{1}{15}$
$= 2$ miles

(21) $$\sum_{n=1}^{\infty} \frac{2}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{2}{n} - \frac{2}{n+1}\right)$$ D
$$= 2 - 1 + 1 - \frac{2}{3} + \frac{2}{3} - \frac{1}{2} + \frac{1}{2} - \frac{1}{5} + \ldots$$
$$= 2$$

(22) $\frac{a+b}{2} = 2\sqrt{ab}$ $\quad \frac{a}{b} = \frac{14}{1}$ D
$a^2 - 14ab + b^2 = 0$
$$a = \frac{14b \pm \sqrt{196b^2 - 4b^2}}{2}$$
$$a = \frac{14b \pm \sim 14b}{2}$$
$$a = 14b$$

23 By definition E

24 $S = \frac{a - ar^n}{1-r}$

$120 = \frac{50 - 50\cdot(\frac{3}{5})^n}{1-\frac{3}{5}}$

$120 = \frac{250 - 250(\frac{3}{5})^n}{2}$

$(\frac{3}{5})^n = \frac{10}{250}$

$n(\log 3 - \log 5) = \log 1 - \log 25$ B

$n(.48 - .7) = 0 - 2(1 - .3)$

$.22n = 1.4$

$n = 6+$

25 $a_1, a_1 - 2, a_1 - 4$, etc 3, 1, −1, −3, −5 −7, . . ., −15 B

$a_2, a_2 r, a_2 r^2$, etc 2, 4, 8, 16, 32, 64, . . ., 1024

$\begin{cases} a_1 + a_2 = 5 \\ a_1 + 2 + a_2 r = 5 \\ a_1 - 4 + a_2 r^2 = 7 \end{cases}$

$\text{Sum}_C = \text{Sum}_A + \text{Sum}_B$

$= -60 + 2046$

$= 1986$

$\cancel{a_1} + a_2 = \cancel{a_1} + 2 + a_2 r$

$a_2 = \frac{2}{r-1}$

$5 - a_2 - 4 + a_2 r^2 = 7$

$1 - \frac{2}{r-1} + \frac{2}{r-1} r^2 = 7$

$r^2 - 3r + 2 = 0$

$(r-2)(r-1) = 0$

$r = 2 \quad r \neq 1$

$a_2 = \frac{2}{2-1} = 2$

$a_1 + 2 = 5$

$a_1 = 3$