1. $s = kmd$

$7 = k \cdot 2 \cdot 3 \Rightarrow k = \frac{7}{6}$

$12 = \frac{7}{6} \cdot 3 \cdot d \Rightarrow d = \frac{24}{7}$

2. The sum of the degree measures is $180(n-2)$, so the mean is

$\frac{180(n-2)}{n} = 170$

$10n = 360$

$n = 36$

3. $5n$

$3m+2$

$n=1, m=1 \Rightarrow 5$ marbles

$5 + 15q > 100 \Rightarrow q = 7$

(15 = LCM of 3 & 5)

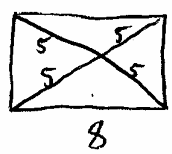
110

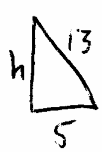
4. $x(30-8) + y(30+18) = (x+y) \cdot 30$

$18y - 8x = 0$

$\frac{x}{y} = \frac{18}{8} = \frac{9}{4}$

5. Any arrangement of SQWAWK & the 20 other ~~arrangements~~ letters is "good", so $P = \frac{21!}{26!}$

6. base: 

diagonal side slice: 

$\Rightarrow h = 12$

$V = \frac{1}{3}bh = \frac{1}{3}(6 \cdot 8) \cdot 12 = 192$

7. To be divisible by 3, its digits must sum to a multiple of 3, so the number of 5's must be a multiple of 3. To be divisible by 5, it must end in a 5 (0 is impossible). 555 is the smallest such number.

8. $\binom{5}{2}(2x)^2\binom{3}{1}(y)^1\binom{2}{2}(3z)^2$

$= 10 \cdot 4x^2 \cdot 3y \cdot 9z^2$

$= 1080x^2yz^2$

9. $(2^2)^{32^j} = (2^4)^{8^j}$

$2^{2\cdot 32^j} = 2^{4\cdot 8^j}$

$2\cdot 32^j = 4\cdot 8^j$

$2\cdot 2^{5j} = 2^2\cdot 2^{3j}$

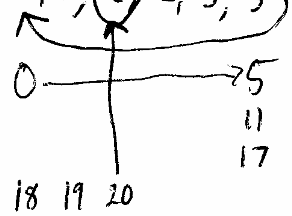
$2^{5j+1} = 2^{3j+2}$

$5j+1 = 3j+2$

$2j = 1$

$j = \frac{1}{2}$

10. $2, 3, \frac{3}{2}, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}$



11.

$R = \frac{2r}{\sqrt{3}} + r$

$R = r\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$

$\frac{r}{R} = \frac{\sqrt{3}}{2+\sqrt{3}} = \frac{\sqrt{3}(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$

$= -3+2\sqrt{3}$

12. $\log_{36} 25 = \log_6 5 = \frac{1}{\log_5 6}$

$= \frac{1}{\log_5 2 + \log_5 3} = \frac{1}{b+c}$

13. You're looking for a number that squares to produce integers & $\sqrt{2}$'s. It will look like $a+b\sqrt{2}$, and its square will be $a^2+2b^2+2ab\sqrt{2}$. So,

$a^2+2b^2 = 67$

$2ab = 42 \Rightarrow ab = 21$

$\Rightarrow a=7, b=3$

$7+3\sqrt{2}$

14. $1\cdot 8^4 = 2^{12} = (16)^3\cdot 1$

$4\cdot 8^2 = 2^8 = (16)^2\cdot 1$

$2\cdot 8 = 16 = 16\cdot 1$

$7\cdot 1 = 7 = 1\cdot 7$

1117_{16}

15. She can only figure it out if every hat she can't see is the same color. Because there are two hats she can't see, their color must be red, so she must see both the black & the green on B & C. $\frac{2}{4\cdot 3} = \frac{1}{6}$

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16. To have 27 zeros at the end, there must be 27 5's in the prime factorization, so

$115 \le N < 120$

117 is a multiple of 3

17. You need an ~~odd~~ number of integers centered on an ~~odd~~ number, or an even number centered on $n+\frac{1}{2}$.

The latter option is possible

$\frac{36}{8} = \frac{9}{2} \Rightarrow 1,2,3,4,5,6,7,8,$

as is the former

$\frac{36}{3} = 12 \Rightarrow 11,12,13$

$\frac{36}{9} = 4 \Rightarrow \cancel{0,1},2\ldots$ 2 ways

18. $n+d=q+7$

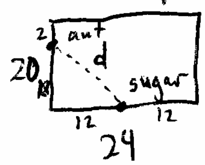
$d=n+q-29$

$n=-n+36$

$2n=36$

$n=18$

19. "unwrap" the glass



$d=\sqrt{12^2+18^2}=6\sqrt{2^2+3^2}=6\sqrt{13}$

20. $\binom{9}{2}=36$ max. possible

8 lines contain 3 points, so were counted 3 times, not 1.

$36-2\cdot 8=20$

21. $3t-4=\sqrt{15t+4}$

$9t^2-24t+16=15t+4$

$9t^2-39t+12=0$

$3t^2-13t+4=0$

$(3t-1)(t-4)=0$

$t=\cancel{\tfrac{1}{3}},4$

22. $x^8y^4 \Rightarrow \binom{12}{8}(1)^8(3)^4x^8y^4$

$=40095x^8y^4$

23. The two parallel lines can't meet, so $\binom{5}{2}-1=9$

24. M must be 1, as it's the "new" digit. A must be 3 so that A+1 = 1 with the help of a carried digit.

25.

6B	0W	1
6W	0B	1
1B	5W	1
5B	1W	1
4B	2W	2
4W	2B	2
3W	3B	2 = 10

26. $10U + T = 3(10T + U) + 5$

$7U = 29T + 5$

$T = 2 \Rightarrow U = 9$

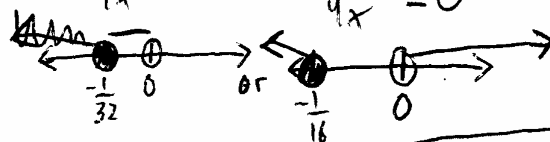
29

27. $g(f(x)) = \sqrt{(f(x)-8)(f(x)-4)}$

$f(x) \geq 8$ or $f(x) \leq 4$

$\frac{-1}{4x} \geq 8$ or $\frac{-1}{4x} \leq 4$

$\frac{-1-32x}{4x} \geq 0$ or $\frac{-1-16x}{4x} \leq 0$



$\left(x \geq -\frac{1}{32} \text{ and } x \neq 0\right)$ or $x \leq -\frac{1}{16}$

28. $k^{-1}(43) = x \Rightarrow k(x) = 43$

$4m^2 + 16m - 33 = 0$

$(2m - 3)(2m + 11) = 0$

$\cancel{m = \frac{3}{2}}$ $m = \frac{-11}{2}$

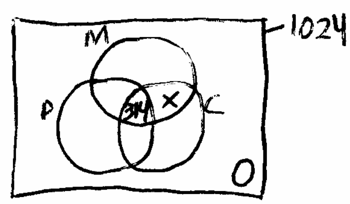
29. $p - r = \frac{4}{4} = 1$

$p + r = \frac{4}{3}$

$-2r = -\frac{1}{3}$

$r = \frac{1}{6}$

30.



$314 + x \leq 512$

$x \leq 198$

31. $\binom{15}{9}(2x^3)^6\left(-\frac{1}{x^2}\right)^9$

$5005 \cdot 64 \cdot -1 = -320{,}320$

32. Let $2x^2-8x-5$ have roots r & s.

$r+s=\frac{-b}{a}=4$

$rs=\frac{c}{a}=-\frac{5}{2}$

x^2+px+q has roots $r+3$ & $s+3$

$-p=r+s+6=4+6 \Rightarrow p=-10$

$q=(r+3)(s+3)=rs+3(r+s)+9$

$=-\frac{5}{2}+3\cdot 4+9=\frac{37}{2}$

$p+q=\frac{-20}{2}+\frac{37}{2}=\frac{17}{2}$

33. $6\frac{1}{2}$th term $=\frac{54}{12}=\frac{9}{2}$

$15\frac{1}{2}$th term $=\frac{180}{30}=\frac{12}{2}$

$\Rightarrow d=\dfrac{\frac{12}{2}-\frac{9}{2}}{15\frac{1}{2}-6\frac{1}{2}}=\frac{1}{6}$

1st term $=\frac{9}{2}+\frac{11}{2}\left(-\frac{1}{6}\right)$

$=\frac{43}{12}$

34. $x=7$ satisfies $3x\equiv 5 \bmod 16$

GCF$(12,16)=4$, so $x\equiv 7 \bmod 12$
$3 \bmod 12$
$11 \bmod 12$

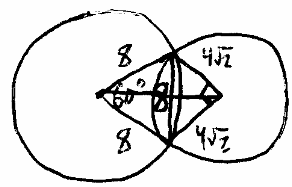
(when you add 16 to x, you add 4 to $x \bmod 12$)

$\overline{21}$

35. Save 750 for last, leaving 500 numbers. 50 end in 0, 1, 2, ... 9, so the sum of the units digits is $50\cdot 45=2250$. Same for the ten's digits. 50 start with 2, ~~4~~ 50 start with 7 & 100 start with 3, 4, 5, 6, adding $9\cdot 50$ & $100\cdot 18$.

$2250+2250+450+1800+\underset{750}{12}=6762$

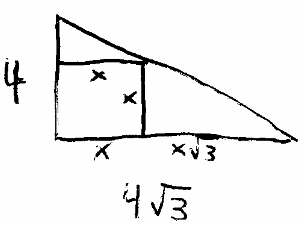
36.



$A=\frac{1}{6}\pi 8^2-\frac{8^2\sqrt{3}}{4}+\frac{\pi}{4}(4\sqrt{2})^2-\frac{1}{2}(4\sqrt{2})^2$

$=\frac{56\pi}{3}$ [illegible] $16\sqrt{3}-16$

37.

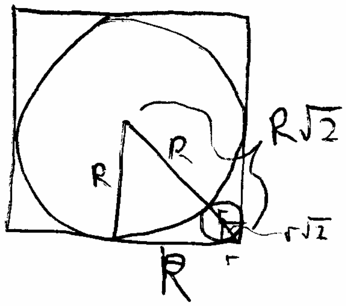


$4\sqrt{3}=x(1+\sqrt{3})$

$x=\frac{4\sqrt{3}}{1+\sqrt{3}}=\frac{4\sqrt{3}(1-\sqrt{3})}{1-3}=6-2\sqrt{3}$

$A=x^2=48-24\sqrt{3}$

38.



$R\sqrt{2} = R + r + r\sqrt{2}$

$R(\sqrt{2}-1) = r(1+\sqrt{2})$

$\frac{r}{R} = \frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{(\sqrt{2}-1)^2}{1} = 3-2\sqrt{2}$

39. $h = 12 \text{ deg/min}$

$m = -30 \text{ deg/min}$

$\frac{360}{h-m} = t_1$ = time between meetings

$= \frac{360}{42} = \frac{60}{7}$

h points up every 30 minutes

m " " " 12 minutes

$LCM(12,30) = 60$

# of meetings $= \frac{60}{\frac{60}{7}} + 1 = 8$

40. $\angle ADE = \frac{77}{2}°$

$\angle OEC = \angle OCE = 20°$ (isosceles $\triangle$)

$\angle ABC = \angle EBD = 180 - 20 - \frac{77}{2}$

$= \frac{320}{2} - \frac{77}{2} = \frac{243}{2}°$