

National Theta Bowl 2000 Answers

1. $\frac{17}{2}$ or $8\frac{1}{2}$ or 8.5
2. -1549
3. 5
4. -1
5. 160
6. 120
7. 6
8. -121
9. 18
10. 8
11. 74
12. 7
13. 21
14. 256
15. 2,985,984

Solutions - National Theta Bowl 2000

$$\begin{aligned} \textcircled{1} \quad -2(4) + 3(3) &= C \\ \underline{C &= 1} \\ -\frac{A}{2} &= \frac{3}{4} \\ \underline{A &= -3/2} \\ -\frac{3}{2}(-2) + 2(3) &= B \\ \underline{B &= 9} \\ A+B+C &= \boxed{8\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad A &= -3 + 49(-4) \\ \underline{A &= -199} \\ B &= 8192(-\frac{1}{2})^{19} \\ B &= 2^{13}(-2)^{-19} \\ \underline{B &= -1/64} \\ C &= \frac{50}{2}[2(-3) + 49(6)] \\ \underline{C &= 7200} \\ D &= \frac{12/5}{1-4/5} = 12 \\ \underline{A+B+C+D} \\ &= -199 - 1350 \\ &= \boxed{-1549} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad A &= 1-2i \\ B &= \frac{1}{1+2i} \cdot \frac{1-2i}{1-2i} \\ B &= \frac{1}{5} - \frac{2}{5}i \\ \underline{C &= -1-2i} \\ D &= \sqrt{1^2+2^2} = \sqrt{5} \\ \underline{ABCD^2} &= -5+10i \\ x+y &= \boxed{5} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad \text{Center of Circle} \\ \underline{(-6, 1)} \\ \text{Center of Hyperbola} \\ \underline{(-2, 4)} \\ \text{Center of Ellipse} \\ \underline{(-5, -3)} \end{aligned}$$

$$\begin{aligned} \underline{\text{Line 2: } m &= 7/3} \\ \text{Line 1: } m &= -3/7 \\ y-1 &= -3/7(x+6) \\ 3x+7y &= -11 \\ \underline{A+B+C} &= \boxed{-1} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad x &= \# \text{ of blue marbles added} \\ \frac{80+x}{200+x} &= \frac{2}{3} \\ \underline{x} &= \boxed{160} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad \text{Roots: } &\left. \begin{array}{l} -1+i\sqrt{3} \\ -1-i\sqrt{3} \end{array} \right\} \\ \text{sum} &= -2 \\ \underline{\text{sum of all roots} &= 3} \\ \Rightarrow 3^{\text{rd}} \text{ root is } &5 \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad \begin{array}{r} 1 \quad -3 \quad a \quad b \\ \hline 5 \quad 10 \quad 5a+50 \\ 1 \quad 2 \quad a+10 \quad b+5a+50 \end{array} \\ b+5a+50 &= 0 \\ \text{product of roots} &= 20 \\ -b &= 20 \Rightarrow b = -20 \\ -20+5a+50 &= 0 \\ a &= -6 \\ \underline{ab} &= \boxed{120} \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad A: 2x+3x+4x &= 180 \\ x &= 20 \\ \underline{A = 4x} &= 80 \end{aligned}$$

$$\begin{aligned} B: \quad \begin{array}{c} \triangle \\ \hline 5 \end{array} \quad B=12 \end{aligned}$$

$$\begin{aligned} C: \quad \sqrt{3e^2} &= \sqrt{12} \\ e &= 2 \\ \underline{V} &= 8 \end{aligned}$$

$$\begin{aligned} D: \quad \begin{array}{c} \triangle \\ \hline x \end{array} \quad \begin{array}{l} 5x^2 = 100 \\ x^2 = 20 \\ \text{Area} = x^2 \end{array} \\ \underline{D} &= 20 \end{aligned}$$

$$\frac{AB}{CD} = \boxed{6}$$

$$\begin{aligned} \textcircled{8} \quad V: A^{-1} &= \frac{1}{31} \begin{bmatrix} 7 & 2 \\ -5 & 3 \end{bmatrix} \\ \underline{\text{sum of entries}} &= 7/31 \end{aligned}$$

$$\begin{aligned} W: AB &= \begin{bmatrix} -15 & 28 \\ 37 & -36 \end{bmatrix} \\ \underline{\text{sum of entries}} &= 14 \end{aligned}$$

$$\begin{aligned} X: B^2 &= \begin{bmatrix} 25 & -36 \\ -54 & 88 \end{bmatrix} \\ \underline{\text{sum of entries}} &= 23 \end{aligned}$$

$$\begin{aligned} Y: \det A &= 31 \\ \underline{Z: \det B} &= -16 \end{aligned}$$

$$\underline{W-X+VY-Z} = \boxed{-121}$$

Solutions - National Theta Bowl 2000

$$(9) A = \log_{1/125} (5^2 \cdot 5^{2/3})$$

$$\left(\frac{1}{125}\right)^A = 5^{8/3}$$

$$5^{-3A} = 5^{8/3}$$

$$-3A = 8/3$$

$$A = -8/9$$

$$\log_{10} 150 = \log_{10} (5^2 \cdot 2 \cdot 3)$$

$$\log_{10} 5 = \log_{10} \left(\frac{10}{2}\right) = 1 - \log 2$$

$$B = 2 \log 5 + \log 2 + \log 3$$

$$B = 2(1 - \log 2) + \log 2 + \log 3$$

$$B = -1 \log 2 + 1 \log 3 + 2$$

$$B = -1x + 1y + 2$$

$$d + e + f = 2$$

$$C: (\log 2)(\log 2x) = (2 \log 2)(\log 4x)$$

$$\log 2x = 2 \log 4x$$

$$\log 2x = \log 16x^2$$

$$2x = 16x^2$$

$$x = 8x^2 \quad (x \neq 0)$$

$$x = 1/8 = C$$

$$D: \log_3 p + \log_3 q = \log_3 pq$$

$$pq = 1/3$$

$$D = \log_3 (1/3) = -1$$

$$\frac{BD}{AC} = \boxed{18}$$

$$(10) x^2 - 8x + 10 = 2x - 11$$

$$x^2 - 10x + 21 = 0 \Rightarrow x = 3 \text{ or } 7$$

$$(3, -5), (7, 3)$$

$$a + b + c + d = \boxed{8}$$

$$(11) A = 16 - 4(3)(-1) = 28$$

$$B = 2^2 - 4 \cdot 2 + 8 = 4$$

$$C: \text{Vertex } (-3, 11)$$

$$C = 8$$

$$D: \text{Focus } (-1, -33/4)$$

$$D = -43/4$$

$$AB + CD = \boxed{74}$$

$$(12) f(1) = 100$$

$$f(2) = \frac{1}{2}(100) = 50$$

$$f(3) = \frac{1}{2}(50) = 25$$

$$f(4) = \frac{1}{2}(25) + \frac{1}{2} = 13$$

$$f(5) = \frac{1}{2}(13) + \frac{1}{2} = \boxed{7}$$

$$(13) b = 32 - 5a$$

$$a = 1 \Rightarrow b = 27$$

$$a = 5 \Rightarrow b = 7$$

$$a = 2 \Rightarrow b = 22$$

$$a = 6 \Rightarrow b = 2$$

$$a = 3 \Rightarrow b = 17$$

$$a = 4 \Rightarrow b = 12$$

$$\text{Sum} = \boxed{21}$$

$$(14) f(1, 2) = 1^2 \cdot 2^1 = 2$$

$$f(4, 2) = 4^2 \cdot 2^4 = \boxed{256}$$

$$(15) \text{possible rational roots}$$

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$$

$$\text{product} = \boxed{2,985,984}$$