THETA SCHOOL TEAM CIPHERING ANSWERS:

1. 2
2. 5
3. 140
4. $\frac{100}{101}$
5. 5
6. 0.236
7. $a=\frac{1}{3}$; b=2
8. 1
9. $(8, \frac{1}{27})$
10. 8
11. $\frac{49977}{50000}$
12. 3
13. -55
14. $\frac{13}{10}$

Theta School Team Cyphering
Solutions

① $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$ — 2

$f(i) = i^5 + i^4 + i^3 + i^2 + i + 1$

$f(-1) = (-i)^5 + (-i)^4 + (-i)^3 + (-i)^2 + (-i) + 1$

$f(i) + f(-i) = 2i^4 + 2i^2 + 2$

$= 2 - 2 + 2$

$= 2$

② — 5

$\begin{cases} x - 2y = 1 \\ 2x - 3y = 3 \end{cases}$

$y = 1$

$x = 3$

$(3, 1)$

$\begin{cases} x - 2y = 1 \\ x - 4y = 9 \end{cases}$

$2y = -8$

$y = -4$

$x = -7$

$(-7, -4)$

$\begin{cases} x - 4y = 9 \\ 2x - 3y = 3 \end{cases}$

$5y = -15$

$y = -3$

$x = -3$

$(-3, -3)$

$$\text{Area}_{\Delta} = \frac{1}{2}\begin{vmatrix} -10 & -5 \\ -4 & -4 \end{vmatrix} = \frac{1}{2}(40 - 30) = 5$$

③ $\begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} =$ — 140

$\begin{bmatrix} 9 & 10 \\ 17 & 22 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} =$

$\begin{bmatrix} 48 & 49 \\ 100 & 105 \end{bmatrix} = 5040 - 4900$

$= 140$

④ $\dfrac{\frac{50(1+199)}{2}}{\frac{50(2+200)}{2}} = \dfrac{200}{202}$ — $\frac{100}{101}$

$= \frac{100}{101}$

⑤ Let $y = \sqrt{x+11} \rightarrow y^2 = x+11$

$\sqrt{x+y} + \sqrt{x-y} = 4$

$x+y+2\sqrt{x^2-y^2}+x-y = 16$

$\sqrt{x^2-y^2} = 8-x$

$x^2-x-y = 64-16x+x^2$

$15x = 75$

$x = 5$

5

⑥ Let $2ab = 4\sqrt{5}$

$ab = 2\sqrt{5}$

$\sqrt{17-4\sqrt{4+4\sqrt{5}+5}}$

$\sqrt{17-4\sqrt{(2+\sqrt{5})^2}}$

$\sqrt{17-8-4\sqrt{5}}$

$\sqrt{9-4\sqrt{5}} = \sqrt{5}-2$

$= 2.236-2$

$= .236$

. 236

⑦ $3\cdot 3^{2x} - 7\cdot 3^x + 2 = 0$

$(3\cdot 3^x - 1)(3^x - 2) = 0$

$3\cdot 3^x = 1 \qquad 3^x = 2$

$x\log 3 = \log \frac{1}{3} \qquad x\log 3 = \log 2$

$x = \frac{\log \frac{1}{3}}{\log 3} \qquad x = \frac{\log 2}{\log 3}$

$x = \log_3 \frac{1}{3} \qquad x = \log_3 2$

$\frac{1}{3} \quad 2$

$\frac{1}{3}, 2$

⑧ $5x = 4 + \frac{2}{1+x}$

$5x^2 + x - 6 = 0$

$(5x+6)(x-1) = 0$

$x \neq -\frac{6}{5} \quad x = 1$

1

⑨ $\begin{cases} \log_2 x^4 - \log_9 y^2 = 15 \\ \log_2 x^{\frac{3}{2}} + \log_9 y^2 = \frac{3}{2} \end{cases}$

$\log_2 x^{\frac{11}{2}} = \frac{33}{2}$

$x^{\frac{11}{2}} = 2^{\frac{33}{2}}$

$x = 2^3 = 8$

$4(3) - \log_3 y = 15$

$\log_3 y = -3$

$y = 3^{-3} = \frac{1}{27}$

$(8, \frac{1}{27})$

(10)

3	1	−5	−2	29	−5	−30
		3	−6	−24	15	30
	1	−2	−8	5	10	0

−1	1	−2	−8	5	10
		−1	3	5	−10
−2	1	−3	−5	10	0
		−2	10	−10	
	1	−5	5	0	

8

$$X^2 - 5X + 5 = 0$$

$$X = \frac{5 \pm \sqrt{5}}{2}$$

$$3 + \frac{5+\sqrt{5}}{2} + \frac{5-\sqrt{5}}{2} = 8$$

(11) The probability that they get at least 2 right is 1 minus the probability that they get 0 right or exactly 1 right. The probability that they get 0 right is the probability that they all fail

$$\therefore \frac{2}{3} \cdot \frac{3}{5} \cdot \frac{4}{4} \cdot \frac{9}{10} = \frac{1}{10}$$

$\frac{1}{10}$ no one works a problem

$\frac{9}{10}$ at least one works the problem

$$0 \text{ right} = {}_5C_0 \left(\frac{9}{10}\right)^0 \left(\frac{1}{10}\right)^5 = \left(\frac{1}{10}\right)^5$$

$$1 \text{ right} = {}_5C_1 \left(\frac{9}{10}\right)^1 \left(\frac{1}{10}\right)^4 = \frac{5 \cdot 9}{10^5}$$

$$\left(\frac{1}{10}\right)^5 + \frac{45}{10^5} = \frac{46}{10^5} = \frac{23}{5 \cdot 10^4}$$

$$1 - \frac{23}{5 \cdot 10^4} = \frac{49977}{50000}$$

$\frac{49977}{50000}$

(12) $\frac{1}{T} + \frac{1}{J} = \frac{1}{\frac{8}{3}}$

$\frac{1}{T} + \frac{1}{J} + \frac{1}{C} = \frac{1}{\frac{12}{7}}$ 3 hrs

$\frac{1}{C} = \frac{7}{12} - \frac{8}{3} = \frac{5}{24}$

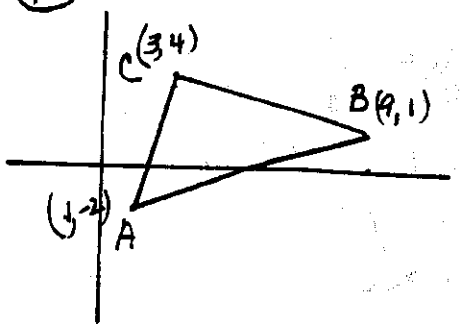
$\frac{1}{4} + \frac{1}{J} = \frac{3}{8}$

$\frac{1}{J} = \frac{1}{8}$

$\frac{1}{J} + \frac{1}{C} = \frac{1}{8} + \frac{5}{24} = \frac{1}{3}$

$\therefore$ 3 hrs

(13) Sketch and "guess" longest side.



$m_{AB} = \frac{3}{8}$ midpt$_{AB}$ $(5, -\frac{1}{2})$ — 55

$m_{\perp} = -\frac{8}{3}$

$8x + 3y = \frac{77}{2}$

$16x + 6y - 77 = 0$

$16 + 6 - 77 = -55$

(14) $\sum_{k=1}^{\infty} \frac{1}{3} \cdot \frac{3^k}{10^k} + \frac{1}{10} \cdot \frac{3^k}{10^k} =$ $\frac{13}{70}$

$\frac{13}{30} \cdot \sum_{k=1}^{\infty} \frac{3^k}{10^k} = \frac{13}{30} \cdot \frac{\frac{3}{10}}{\frac{7}{10}} = \frac{13}{70}$