1992 National Mu Alpha Theta Convention

Radicals Answer Key:

1. D
2. C
3. A
4. E
5. D
6. B
7. C
8. C
9. B
10. D
11. B
12. A
13. B
14. D
15. C
16. C
17. A
18. C
19. A
20. B
21. E
22. B
23. B
24. A
25. D
26. B
27. B
28. C
29. C
30. D

Radicals - Topic Test

D 1. $\sqrt{2}$ is an infinite, nonrepeating decimal (by definition).

C 2. $x = \sqrt{27} + \sqrt{48}$

$x = 3\sqrt{3} + 4\sqrt{3}$

$x^2 = (3\sqrt{3} + 4\sqrt{3})^2$

$x^2 = 27 + 72 + 48$

$\boxed{x^2 = 147}$

A 3. $\dfrac{\sqrt{72} + \sqrt{108}}{\sqrt{8} + \sqrt{12}}$

$\dfrac{6\sqrt{2} + 6\sqrt{3}}{2\sqrt{2} + 2\sqrt{3}}$

$\dfrac{6\cancel{(\sqrt{2}+\sqrt{3})}}{2\cancel{(\sqrt{2}+\sqrt{3})}}$

3

E 4. $\left(\sqrt{2\sqrt{2\sqrt{4}}}\right)\left(\sqrt{3\sqrt{3\sqrt{9}}}\right)$

$\left(\sqrt{2\sqrt{2\cdot 2}}\right)\left(\sqrt{3\sqrt{3\cdot 3}}\right)$

$\left(\sqrt{2\cdot 2}\right)\left(\sqrt{3\cdot 3}\right)$

$(2)\ (3)$

6

D. 5. $\frac{2}{\sqrt{6}}x = 3\sqrt{2}$

$2x = 3\sqrt{12}$

$2x = 6\sqrt{3}$

$\boxed{x = 3\sqrt{3}}$

B 6. $\sqrt{2} = n\%(4\sqrt{2})$

$\frac{\cancel{\sqrt{2}}}{4\cancel{\sqrt{2}}} = n\%$

$\frac{1}{4} = n\%$

$\boxed{25\%}$

C 7. $\sqrt{12} = N$ $N = 3.46$

$3 < \sqrt{12} < 4$

Try $3.4 \times 3.4 = 11.56$
$3.5 \times 3.5 = 12.25$ ⇒ Try $3.44 \times 3.44 = 11.8336$
$3.45 \times 3.45 = 11.9025$
$3.46 \times 3.46 = 11.9716$
$3.47 \times 3.47 = 12.0409$

$\boxed{(3.46)^2 \text{ is closer to } 12.}$

C 8. $|7+24i| = \sqrt{7^2+24^2}$

$= \sqrt{49+576}$

$= \sqrt{625}$ 25

B. 9. $\dfrac{3}{2\sqrt[3]{3} + \sqrt[3]{5}}$

$2^3 \cdot 3 + 5 = 8 \cdot 3 + 5 = 24 + 5 = \boxed{29}$

D. 10. $\dfrac{-2 \pm \sqrt{7}}{3}$

Sum: $\frac{-2+\sqrt{7}}{3} + \frac{-2-\sqrt{7}}{3} = \frac{-4}{3} = \frac{-b}{a}$

Product: $\frac{4-7}{9} = \frac{-3}{9} = \frac{-1}{3} = \frac{c}{a}$

$ax^2 + bx + c = 0$

$3x^2 - 4x - 1 = 0$

$a + b - c$

$3 - 4 + 1$

2

Radicals – Topic Test

(3, x) (5, 3) p.c.

(B) 11. $\sqrt{2\sqrt{63} + \frac{2(8-3\sqrt{7})}{(8+3\sqrt{7})(8-3\sqrt{7})}}$

$\sqrt{6\sqrt{7} + \frac{2(8-3\sqrt{7})}{64-63}}$

$\sqrt{6\sqrt{7} + 16 - 6\sqrt{7}}$

$\sqrt{6\sqrt{7} - 6\sqrt{7} + 16}$

$\sqrt{16}$

$\boxed{4}$

(A) 12. $\frac{\sqrt[3]{320} \cdot \sqrt[3]{200}}{\sqrt[4]{1800} \cdot \sqrt[4]{450}}$

$\frac{\sqrt[3]{2^6 \cdot 5} \cdot \sqrt[3]{2^3 \cdot 5^2}}{\sqrt[4]{2^3 3^2 \cdot 5^2} \ \sqrt[4]{3^2 \cdot 5^2 \cdot 2}}$

$\frac{\sqrt[3]{2^9 \cdot 5^3}}{\sqrt[4]{2^4 \cdot 3^4 \cdot 5^4}}$

$\frac{\cancel{2} \cdot 2 \cdot 2 \cdot \cancel{5}}{\cancel{2} \cdot 3 \cdot \cancel{5}}$

$\boxed{\frac{4}{3}}$

Answer

$2(4) + 3 = 8 + 3 = 11$

(B) 13. $x = \sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \cdots}}}}$

$x^2 = 12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \cdots}}}$

$x^2 = 12 + x$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x - 4 = 0 \quad x + 3 = 0$

$x = 4 \quad x = -3$

$\boxed{x = 4}$

(D) 14. $\sqrt{(3-5)^2 + (x-3)^2} = (2\sqrt{17})^2$

$\sqrt{4 + (x^2 - 6x + 9)} = 2\sqrt{17}$

$x^2 - 6x + 13 = 68$

$x^2 - 6x + 13 - 68 = 0$

$x^2 - 6x - 55 = 0$

$(x-11)(x+5) = 0$

$x - 11 = 0 \quad x + 5 = 0$

$x = 11 \quad x = -5$

$11 + (-5) = 6$

(C) 15. $\sqrt{6}x^2 - 4x - 2\sqrt{6} = 0$

Either:

$\frac{-b}{a} = \frac{4}{\sqrt{6}} = \frac{4\sqrt{6}}{6} = \boxed{\frac{2\sqrt{6}}{3}}$ $\boxed{\text{or}}$ quadratic formula

$x = \frac{4 \pm \sqrt{16+48}}{2\sqrt{6}} = \frac{4 \pm \sqrt{64}}{2\sqrt{6}} = \frac{4 \pm 8}{2\sqrt{6}}$

$x = \frac{4+8}{2\sqrt{6}} = \frac{12}{2\sqrt{6}} = \frac{12\sqrt{6}}{2 \cdot 6} = \frac{12\sqrt{6}}{12} = \sqrt{6}$

$x = \frac{4-8}{2\sqrt{6}} = \frac{-4}{2\sqrt{6}} = \frac{-4\sqrt{6}}{12} = \frac{-\sqrt{6}}{3}$

$\frac{3\sqrt{6}}{3} - \frac{\sqrt{6}}{3} = \boxed{\frac{2\sqrt{6}}{3}}$

(C) 16. $\left(2^{1+\sqrt{2}}\right)^{1-\sqrt{2}} + \left(\frac{8}{27}\right)^{-\frac{2}{3}}$

$2^{1-2} + \left(\frac{27}{8}\right)^{\frac{2}{3}}$

$2^{-1} + \frac{3^2}{2^2}$

$\frac{1}{2} + \frac{9}{4}$

$\frac{2}{4} + \frac{9}{4}$

$\boxed{\frac{11}{4}}$

Radical - Topic Test.

A) 17.

$$R = \frac{SK}{T} \qquad R = \frac{2S}{T}$$

$$\frac{4}{3} = \frac{\frac{3}{7}K}{\frac{9}{14}} \qquad \sqrt{48} = \frac{2S}{\sqrt{75}}$$

$$\frac{\cancel{4}^{2}}{\cancel{3}} \cdot \frac{\cancel{9}}{\cancel{14}_{2}} \cdot \frac{\cancel{7}^{1}}{\cancel{3}} = K \qquad 4\sqrt{3} = \frac{2S}{5\sqrt{3}}$$

$$2 = K \qquad 60 = 2S$$

$$\boxed{30 = S}$$

B) 20.

$$\sqrt[3]{\frac{\sqrt{112} - \sqrt{9072}}{\sqrt{112}}} - \sqrt[4]{\frac{\sqrt{1008} - \sqrt{448}}{\sqrt{112}}}$$

$$\sqrt[3]{1 - \sqrt{81}} - \sqrt[4]{\sqrt{9} - \sqrt{4}}$$

$$\sqrt[3]{1 - 9} - \sqrt[4]{3 - 2}$$

$$\sqrt[3]{-8} - \sqrt[4]{1}$$

$$-2 - 1$$

$$-3$$

C) 18.

$$\sqrt[3]{x^3 + 6x^2 - 4} = x + 2$$

$$x^3 + 6x^2 - 4 = (x+2)^3$$

$$\cancel{x^3} + \cancel{6x^2} - 4 = \cancel{x^3} + \cancel{6x^2} + 12x + 8$$

$$-12 = 12x$$

$$-1 = x$$

one solution only.

E) 21.

$$\sqrt{2},\ \sqrt[3]{2},\ \sqrt[6]{2}$$

$$2^{\frac{1}{2}},\ 2^{\frac{1}{3}},\ 2^{\frac{1}{6}}$$

$$r = \frac{2^{\frac{1}{3}}}{2^{\frac{1}{2}}} = 2^{\frac{1}{3} - \frac{1}{2}} = 2^{-\frac{1}{6}}$$

Thus, the fourth term

$$2^{\frac{1}{6}} \cdot r = 2^{\frac{1}{6}} \cdot 2^{-\frac{1}{6}} = 2^{0} = \boxed{1}$$

A) 19.

$$\left(\sqrt{3\sqrt[3]{2}}\right)\left(\sqrt{2\sqrt[3]{3}}\right)\left(\sqrt[3]{2\sqrt{3}}\right)\left(\sqrt[6]{3}\right)$$

$$\left(\left(3 \cdot 2^{\frac{1}{3}}\right)^{\frac{1}{2}}\right) \quad \left(\left(2 \cdot 3^{\frac{1}{3}}\right)^{\frac{1}{2}}\right) \quad \left(\left(2 \cdot 3^{\frac{1}{2}}\right)^{\frac{1}{3}}\right) \quad \left(3^{\frac{1}{6}}\right)$$

$$\left(3^{\frac{1}{2}} 2^{\frac{1}{6}}\right) \quad \left(2^{\frac{1}{2}} \cdot 3^{\frac{1}{6}}\right) \quad \left(2^{\frac{1}{3}} \cdot 3^{\frac{1}{6}}\right)$$

$$\left(3^{\frac{3}{6}} 2^{\frac{1}{6}}\right) \quad \left(2^{\frac{3}{6}} \cdot 3^{\frac{1}{6}}\right) \quad \left(2^{\frac{2}{6}} 3^{\frac{1}{6}}\right)$$

$$\left(\sqrt[6]{3^3 2}\right) \quad \left(\sqrt[6]{2^3 \cdot 3}\right) \quad \left(\sqrt[6]{2^2 3^1}\right) \quad \left(\sqrt[6]{3}\right)$$

$$\left(\sqrt[6]{3^3 \cdot 2 \cdot 2^3 \cdot 3 \cdot 2^2 \cdot 3 \cdot 3}\right)$$

$$\sqrt[6]{3^6 \cdot 2^6}$$

$$3 \cdot 2$$

$$6$$

B) 22.

$$x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}}$$

$$x^2 = 1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}$$

$$x^2 = 1 + x$$

$$x^2 - x - 1 = 0$$

$$x = \frac{1 \pm \sqrt{1^2 - 4(1)(-1)}}{2}$$

$$x = \frac{1 \pm \sqrt{1 + 4}}{2}$$

$$\sqrt{5} \doteq 2.236 \qquad x = \frac{1 \pm \sqrt{5}}{2}$$

$$x = \frac{1 \pm 2.24}{2}$$

positive value.

$$x = \frac{1 + 2.24}{2}$$

$$x = \frac{3.24}{2}$$

$$\boxed{x = 1.62}$$

(B) 23. $x - 11 = 2\sqrt{x+4}$

$x^2 - 22x + 121 = 4(x+4)$

$x^2 - 22x + 121 = 4x + 16$

$x^2 - 26x + 105 = 0$

$(x-21)(x-5) = 0$

$x - 21 = 0 \quad x - 5 = 0$

$x = 21 \quad \cancel{x = 5}$ extraneous.

$\boxed{\text{1 solution}}$

(A) 24. $\frac{3}{\sqrt{y}} - \frac{1}{\sqrt{x}} = \frac{2}{\sqrt{x}+\sqrt{y}}$

$\frac{3\sqrt{x} - \sqrt{y}}{\sqrt{xy}} = \frac{2}{\sqrt{x}+\sqrt{y}}$

$3x - \sqrt{xy} + 3\sqrt{xy} - y = 2\sqrt{xy}$

$3x + 2\sqrt{xy} - y = 2\sqrt{xy}$

$3x - y = 0$

$3x = y$

$x = \frac{y}{3}$

$\frac{x}{y} = \frac{\frac{y}{3}}{y}$

$\frac{x}{y} = \frac{\cancel{y}}{3} \cdot \frac{1}{\cancel{y}}$

$\frac{x}{y} = \boxed{\frac{1}{3}}$

(D) 25. $\sqrt{x+8} - \frac{5}{\sqrt{x+8}} = 4$

$x + 8 - 5 = 4\sqrt{x+8}$

$x + 3 = 4\sqrt{x+8}$

$x^2 + 6x + 9 = 16(x+8)$

$x^2 + 6x + 9 = 16x + 128$

$x^2 - 10x - 119 = 0$

$(x+7)(x-17) = 0$

$x + 7 = 0 \quad x - 17 = 0$

$\boxed{\cancel{x = -7} \text{ extraneous}} \quad \boxed{x = 17 \text{ true root}}$

$\boxed{\begin{array}{l} x = 17 \text{ true} \\ x = -7 \text{ extraneous} \end{array}}$

(B.) 26. $\left(\sqrt[3]{\sqrt{45} + \sqrt{20}}\right)^{-2}$

$\left(\sqrt[3]{3\sqrt{5} + 2\sqrt{5}}\right)^{-2}$

$\left(\sqrt[3]{5\sqrt{5}}\right)^{-2}$

$\left(\sqrt[3]{\sqrt{125}}\right)^{-2}$

$\left(5^{\frac{3}{2}}\right)^{-\frac{2}{3}}$

5^{-1}

$\frac{1}{5}$

(B) 27. $\sqrt{3-2x} + \sqrt{8x-7} = \sqrt{6x-2}$

$3 - 2x + 8x - 7 + 2\sqrt{(3-2x)(8x-7)} = 6x - 2$

$6x - 4 + 2\sqrt{-16x^2 + 3x - 21} = 6x$

$2\sqrt{-16x^2 + 38x - 21} = 2$

$\sqrt{-16x^2 + 38x - 21} = 1$

$-16x^2 + 38x - 21 = 1$

$16x^2 - 38x + 21 = -1$

$16x^2 - 38x + 22 = 0$

$8x^2 - 19x + 11 = 0$

$(8x - 11)(x - 1) = 0$

$8x - 11 = 0 \quad x - 1 = 0$

$8x = 11 \quad x - 1 = 0$

$x = \frac{11}{8} \quad x = 1$

$\frac{11}{8} - 1 = \frac{3}{8}$

(C) 28. $\sqrt[n]{\dfrac{4^{2n+1}+2^{4n+1}}{6}}$

$\sqrt[n]{\dfrac{2^{4n+2}+2^{4n+1}}{6}}$

$\sqrt[n]{\dfrac{2^{4n+1}(2+1)}{6}}$

$\sqrt[n]{\dfrac{2^{4n}(2^1)(3)}{6}}$

$\sqrt[n]{2^{4n}}$

$2^{4n\cdot\frac{1}{n}}$

2^4

16

(C) 29. $\sqrt{1+x}-\sqrt{1-x}\geq\sqrt{x}$

Solve as an equation; determine and test boundaries.

$\sqrt{1+x}\geq\sqrt{x}+\sqrt{1-x}$

$\sqrt{1+x}=\sqrt{x}+\sqrt{1-x}$

$1+x = x+1-x+2\sqrt{x}\sqrt{1-x}$

$1+x = 1+2\sqrt{x-x^2}$

$x = 2\sqrt{x-x^2}$

$x^2 = 4(x-x^2)$

$x^2 = 4x-4x^2$

$0 = 4x-5x^2$

$4x-5x^2=0$

$x(4-5x)=0$

$x=0 \quad 4-5x=0$

$-5x=-4$

$x=\frac{4}{5}$

Since:

$\sqrt{1+x}\geq 0$

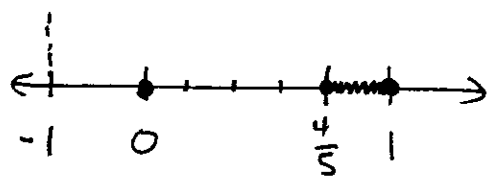
$1+x\geq 0$

$x\geq -1$

$\sqrt{1-x}\geq 0$

$1-x\geq 0$

$1\geq x$



THE SOLUTION MUST BE BETWEEN -1 and 1.

TEST $\frac{1}{5}$

$\sqrt{1+\frac{1}{5}}-\sqrt{1-\frac{1}{5}}\geq\sqrt{\frac{1}{5}}$

$\sqrt{\frac{6}{5}}-\sqrt{\frac{4}{5}}\geq\sqrt{\frac{1}{5}}$

$\dfrac{\sqrt{30}-2\sqrt{5}}{5}\not\geq\dfrac{\sqrt{5}}{5}$

false.

values less than zero will yield complex roots.

a represents the largest root 1

b " " the smallest root 0

$\boxed{1+0=1}$

radicals - topic test p. 6.

d.) 30.

$$\begin{array}{r|rrrrr} 1+\sqrt{3} & 1 & 3\sqrt{3} & -6-11\sqrt{3} & 3+12\sqrt{3} & 2-4\sqrt{3} \\ \hline & & 1+\sqrt{3} & 13+5\sqrt{3} & -11+\sqrt{3} & 31+5\sqrt{3} \\ \hline & 1 & 1+4\sqrt{3} & 7-6\sqrt{3} & -8+13\sqrt{3} & 33+\sqrt{3} \end{array}$$

$e+f\sqrt{3} = 33+\sqrt{3}$

$e+f = 33+1$
$= \boxed{34}$