Probability Topic Test Answers:

1. A
2. E
3. D
4. C
5. A
6. E
7. C
8. D
9. B
10. E
11. B
12. ~~E~~ out
13. D
14. C
15. C
16. B
17. A
18. C
19. B
20. A
21. B
22. D
23. E
24. B
25. B

Probability Topic Test
Solutions — Nationals '91

① 1 favorable, 5 possibilities
$\therefore P(\text{correct answer}) = \frac{1}{5}$ A

② ${}_{10}C_7 = {}_{10}C_3 = \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} = \frac{10!}{7!3!}$ E

③ $\binom{7}{5} + {}_{10}P_4 = \frac{7 \cdot 6}{1 \cdot 2} + 10 \cdot 9 \cdot 8 \cdot 7$ D

$= 21 + 5040$

$= 5061$

④ $\frac{7!}{4!} = 5 \cdot 6 \cdot 7 = 210$ C

⑤ Scores 2, 3, 4, or 5 on each hole
Possibilities
18 - 2's = 36
17 - 2's + 1 - 3 = 37
15 - 2's, 1 - 3 + 1 - 5 = 38
etc
It is possible to score every value between 39 and 90 incl.
$\therefore$ 55 possible scores A

⑥ Primes 2, 3, 5, 7 | 4 | 4 | 4 | 4 | E

⑦ $P(\ell) = {}_4C_2 \left(\frac{3}{10}\right)^2 \left(\frac{7}{10}\right)^2$ C

$= 6 \cdot \frac{9}{100} \cdot \frac{49}{100}$

$= \frac{2646}{(100)^2}$

$= .2646$

⑧

	h	l	t
h	hh	(hl)	(ht)
l	(lh)	ll	(lt)
t	(th)	(tl)	tt

$\frac{6}{27} = \frac{2}{9}$ D

⑨ A wins or B wins ⟶ other horse loses B

$\frac{5}{6} + \frac{1}{3} = \frac{8}{9}$

Prob. other horse wins: $(1 - \frac{8}{9}) = \frac{1}{9}$

Odds are $\frac{1}{8}$

⑩ Total ways to fall 20^2 E

Way to roll 32: 7+25 ... 25+7

Which gives 19 ways

Prob. of rolling a sum of 32: $\frac{19}{400}$

⑪ To get an odd sum must have B

1 even + 1 odd.

Probability of odd sum: $\frac{{}_{50}C_1 \cdot {}_{50}C_1}{{}_{100}C_2} = \frac{50 \cdot 50}{\frac{100 \cdot 99}{2}}$

$= \frac{50}{99}$

⑫ $P((A \cap B) \cup C) = P(A \cap B) + P(C) - P((A \cap B) \cap C)$ E

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A \cap B) = \frac{1}{4} + \frac{3}{11} - \frac{1}{10} = \frac{93}{220}$

$P((A \cap B) \cup C) = \frac{93}{220} + \frac{2}{5} - \frac{1}{5}$

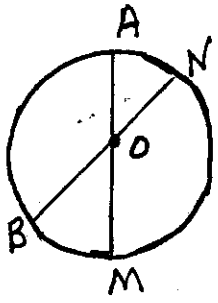
$= \frac{137}{220}$

⑬ 3 A

4 G Circular $(4-1)! = 3!$ D

2 F $3!\,3!\,4!\,2!\,3! =$

3 R $6 \cdot 6 \cdot 24 \cdot 2 \cdot 6 = 10368$

(14) 

Place Pt. A and Pt. B anywhere on Circle O. Draw diameters AM + BN. Pt. C can be placed anywhere, but if it is on arc $\overset{\frown}{MN}$, it will not be on the semicircle. $\overset{\frown}{MN}$ could be $\frac{1}{4}$ of the circle. C

$\therefore \frac{3}{4}$ is probability that all 3 are on a semicircle.

(15) Expected value is probability times amount you win.

$TT = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ $\quad \frac{1}{4}(10) + \frac{1}{2}(5) + \frac{1}{4}(1)$ C

$TH = \frac{1}{2}$

$HH = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ $\quad 2.50 + 2.50 + .25 = 5.25$

(16) Formula for no matches

$$P_{[0]} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \ldots \pm \frac{1}{N!}$$ B

$$P_{[0]} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}$$

$$= \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \frac{1}{720}$$

$$= \frac{360 - 120 + 30 - 6 + 1}{720}$$

$$= \frac{53}{144}$$

You can also manually count the ways to match and then subtract from 720.

B

1 way for all 6 to match
0 ways for ex. 5 to match
15·1 ways for ex. 4 to match
20·2 ways for ex. 3 to match
15·9 ways for ex 2 to match
6·44 ways for ex 1 to match

1 + 15 + 40 + 135 + 264 = 455 ways to match

720 − 455 = 265 ways not to match

$\therefore \frac{265}{720} = \frac{53}{144}$

(17) Must go all way around clockwise or counterclockwise

$\frac{1}{3}\cdot\frac{3}{4}\cdot\frac{1}{2} + \frac{2}{3}\cdot\frac{1}{4}\cdot\frac{1}{2} = \frac{5}{24}$

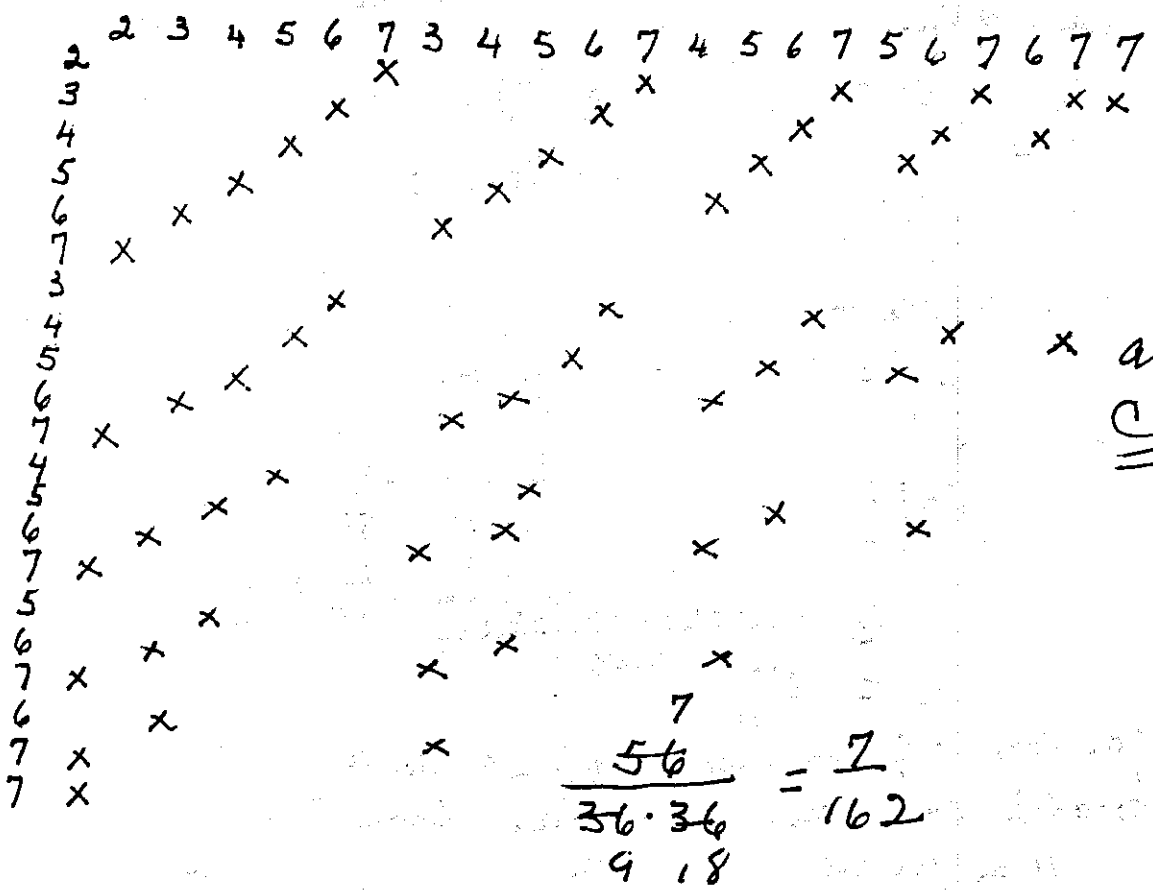
#17 A

alternate sol. you could count them by knowing
1 two 6 seven 6
2 three 5 eight 10
3 four 4 five 12
28

Double $\frac{56}{36\cdot 36}$

(18)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12



ans. C

7

$\frac{56}{36\cdot 36} = \frac{7}{162}$

9 18

(19) 2A 3O 4P

1st draw $\frac{1}{3}$

2nd draw with replacement $\frac{1}{2}\cdot\frac{1}{3}$

or 2nd draw without replacement $\frac{1}{2}\cdot\frac{1}{4}$ B

$\frac{1}{3}\cdot\frac{1}{6} + \frac{1}{3}\cdot\frac{1}{8} =$

$\frac{1}{18} + \frac{1}{24} = \frac{7}{72}$

(20) $\dfrac{\frac{8!}{3!5!}}{\frac{16!}{8!8!}} = \dfrac{{}_8C_3}{{}_{16}C_8}$ A

(21) By definition B
Components sum to 1. B

(22) Select cage B and Pick a shipment A

$\frac{1}{3} \cdot \frac{4}{8} = \frac{1}{6}$

(23) A $-2R+1G$ B $-3R+2G$

Prob 2 reds $\frac{1}{3}$ $5R+2G$

Prob 1 red + 1 green $\frac{2}{3}$ $4R$ and $3G$ E

Drawing $1R+1G$ from B

$\frac{1}{3} \cdot \frac{{}_5C_1 \cdot {}_2C_1}{{}_7C_2} = \frac{1}{3} \cdot \frac{10}{21} = \frac{10}{63}$

$\frac{2}{3} \cdot \frac{{}_4C_1 \cdot {}_3C_1}{{}_7C_2} = \frac{2}{3} \cdot \frac{12}{21} = \frac{24}{63}$

$\frac{10}{63} + \frac{24}{63} = \frac{34}{63}$

(24) consider one of spades + card of diamonds as 1 unit B

$\therefore \frac{2 \cdot 51!}{52!} = \frac{2 \cdot 51!}{2 \cdot 52 \cdot 51!} = \frac{1}{26}$

(25) Chartreuse Pink

1st	$\frac{1}{3}$	$\frac{2}{3}$	PCCC	$\frac{2}{3} \cdot \frac{1}{2} \cdot \frac{3}{5} \cdot \frac{2}{3} = \frac{2}{15}$
2nd	$\frac{1}{2}$	$\frac{1}{2}$	CPCC	$\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{3}{5} \cdot \frac{2}{3} = \frac{1}{15}$
3	$\frac{3}{5}$	$\frac{2}{5}$	CCPC	$\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{5} \cdot \frac{2}{3} = \frac{2}{45}$
4th	$\frac{2}{3}$	$\frac{1}{3}$	CCCP	$\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{3}{5} \cdot \frac{1}{3} = \frac{1}{30}$

B

$\frac{2}{15} + \frac{1}{15} + \frac{2}{45} + \frac{1}{30} = \frac{25}{90} = \frac{5}{18}$