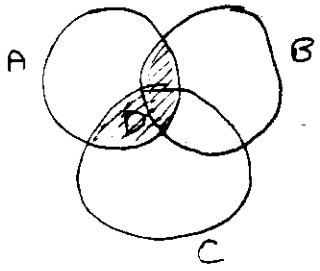
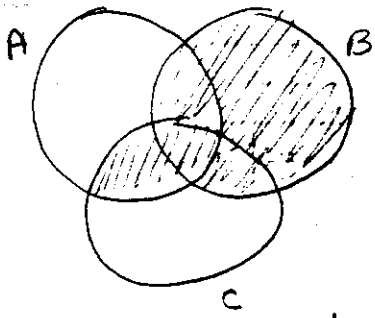
1. $140 \div 7$ A $360 \div 9$ D $319 \div 11$ C $1078 \div 11 = 7$ E

B

2. $22 \cdot 21 \cdot 20 \cdot \ldots \cdot 15 \cdots 10 \cdots 5$ 4 A $(22!)^2$ original question

3. 



$B \cup (A \cap C) = D \cup B$ C

4. must be some number more than a common multiple of all three

$4 \cdot 5 \cdot 9 = 180$ 360 540 720 722 E

5. 4 4 3 2 96 C

6. $2700 = 2^2 \cdot 3^3 \cdot 5^2$ # of divisors $= 3 \cdot 4 \cdot 3 = 36$ 3 A

7. remainder three when divided by 6 is $6k+3 = 3(2k+1)$ which cannot be prime unless $k=0$ which is not between 50 + 100 answer is a A

8. abundant - sum of proper factors > #
perfect - sum of proper factors = #

X 2 3 4 ~~9~~ 6 = 1+2+3 perfect 8 1,2,4 ~~9~~ 1,3

~~10~~ 1,2,5 ~~11~~ 12 < 1,2,3,4,6 abundant ~~13~~ ~~14~~ ~~15~~ ~~16~~ 1,2,4,8

~~17~~ 18 < 1,2,3,6,9 abundant ~~19~~ 20 < 1,2,4,5,10 abundant 28 = 1,2,4,7,14 perfect

$6 + 12 + 18 + 20 + 28 = 84$ E

9.

96	165	627	1353	4 C
69	561	726	3531	
165	726	1353	4884	

10. $x = 1 + 11t$, $y = 3 - 2t$ $t=0$ $(1,3)$ $t=1$ $(12,1)$ 2 B

11. 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

1597, 2584, 4181 **E**

12. $999 - 99 = 900$ 3-digit numbers

three digit numbers without fives

8·9·9

$81 \times 8 = 648$

$900 - 648 = 252$ **B**

13. $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3$

$\left[\frac{n(n+1)}{2}\right]^2 \quad \left[\frac{10(11)}{2}\right]^2 = (55)^2$

55 × 55: 275, 2750 → 3025

$3025\ cm^3 = 3025\ ml = 3.025\ l$

$\frac{3.025}{2} = 1.5125$ 2 liters **B**

14. $m = n+2$

$18 + 2m = 3n - 7$

$18 + 2(n+2) = 3n - 7$

$18 + 2n + 4 = 3n - 7$

$29 = n$

$31 = m$

$60(2) = 120$ **D**

$18 + 2(n-2) = 3n - 7$

$14 + 2n = 3n - 7$

$14 = n$

$19 = m$

$40(2)$

15. $5b + 4 = 3(b+6)$

$2b = 14$

$b = 7$

$25 = 3(7) + 4$ $\quad 25_{10} = 34_7$ **D**

16. $2|5400$, $2|2700$, $2|1350$, 675

$3|675$, $3|225$, $3|75$, 25

$2^3 \cdot 3^3 \cdot 5^2$

$(1+2+4+8)(1+3+9+27)(1+5+25)$

$(15)(40)(31)$

$(600)(31)$

18600 **D**

31

17. 3, 5, 9, 17, 32 **58** **100**

2 4 8 15 26 **42** **D**

2 4 7 11 16

2 3 4 5

18. $\frac{\frac{2}{8} \cdot 8^3}{\left(1 - \frac{1}{8^3}\right) \cdot 8^3} + \frac{\frac{3}{8^2} 8^3}{\left(1 - \frac{1}{8^3}\right) \cdot 8^3} = \frac{2(8^2)}{511} + \frac{24}{511} = \frac{152}{511}$ **E**

Ⓐ | Ⓑ | Ⓒ | Ⓓ | Ⓔ

9. 8 divides $2m-3$ means $2m-3 = 8k$ k is an integer

$m = \frac{8k+3}{2}$

Ⓐ $2\left(\frac{8k+3}{2}\right)+3 = 8k+6$ not divisible by 8

Ⓑ $2\left(\frac{8k+3}{2}\right)-8 = 8k-5$ not divisible by 8

Ⓒ $4\left(\frac{8k+3}{2}\right)+8 = 16k+14$ not divisible by 8

Ⓓ $5\left(\frac{8k+3}{2}\right)+1 = \frac{40k+15}{2}+1 = 20k+\frac{17}{2}$ not divisible by 8

Ⓔ $6\left(\frac{8k+3}{2}\right)-9 = 24k+9-9 = \underline{24k}$! **E**

20. $A = 3! + 4! + 5! + 6! + 7! + \ldots + 15!$

$6 + 24 + 120 + 720 + \ldots$

$= (150) + 240k$ for some integer k **B**

21. Ⓐ $n^3 - n - 6n + 3 = \underbrace{n(n-1)(n+1)}_{\text{one of these is a multiple of 3.}} - 3(2n+1)$ **A**

B. $n=1$ -4 not divisible by 3

C. $n=3$ 8 " " " "

D. $n=1$ -1 " " " "

E. $n=2$ 2 " " " "

22. Divisible by 11 odd digits − even digits is divisible by 11. No way to get odds − evens = 11 must be odds − evens = 0.

7469, 7964, 6479, 6974, 9647, 9746, **B**

4697, 4796 8

23.

0^2	1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2
0	1	4	9	16	25	36	49	64	81	100

remainders = 0, 1, 4, 5, 3, 7 not 2 or 6

$8k+3 - (8k+1)$ $R = 2$ not a choice

$8k+1 - (8k+3)$ $R = -2$ or 6 **D**

24. $\frac{m \cdot 9!}{9!} = \frac{151 \cdot \overset{5}{\cancel{150}} \cdot 149 \cdot \overset{74}{\cancel{148}} \cdot \overset{2}{\cancel{147}} \cdot \overset{73}{\cancel{146}} \cdot 145 \cdot \overset{2}{\cancel{144}} \cdot 143}{9 \cdot 8 \cdot 7 \cdot \overset{3}{\cancel{6}} \cdot 5 \cdot \overset{2}{\cancel{4}} \cdot 3 \cdot 2 \cdot 1}$ **E**

$m = (151) \cdot 5 \cdot 149 \cdot 74 \cdot 7 \cdot 73 \cdot 145 \cdot 143$

prime

25. $1999 \equiv 1 \bmod 9$

$(1999)^{1999} \equiv 1^{111} \bmod 9$ **B**

26. $m^2 + a$ is not a multiple of 10 - all possible unit's digits

0 1 4 9 16 25 36 49 64 81 100

0 1 4 5 6 9 a can't be 0, 1, 9, 4, 6, 5

a can be 2, 3, 7, 8 **20**

27. $\frac{1}{48} + \frac{1}{36} = \frac{3+4}{2\cdot2\cdot2\cdot2\cdot3\cdot3} = \frac{7}{144}$ LCM 1008 **D**

$48 = 2\cdot2\cdot2\cdot2\cdot3$ $36 = 2\cdot2\cdot3\cdot3$

28. deficient 11, 13, 14, 15, 16, 17, 19, 21, 22, 23, 25, 26, 27, 29

12 → 1, 2, 3, 4, 6, 20 → 1, 2, 4, 5, 10 30 → 1, 2, 3, 5, 6, 10, 15

18 → 1, 3, 6, 9 24 → 1, 2, 3, 4, 6, 8, 12

$P(\text{deficient}) = \frac{14}{20} = 0.7$ **E**

29. K odd $K(K-1)(K+1)$

{ K+1 even
{ K-1 even
one is multiple of 4
other must be multiple of 2
one of the three is a multiple of 3

$2 \cdot 3 \cdot 4 = 24$

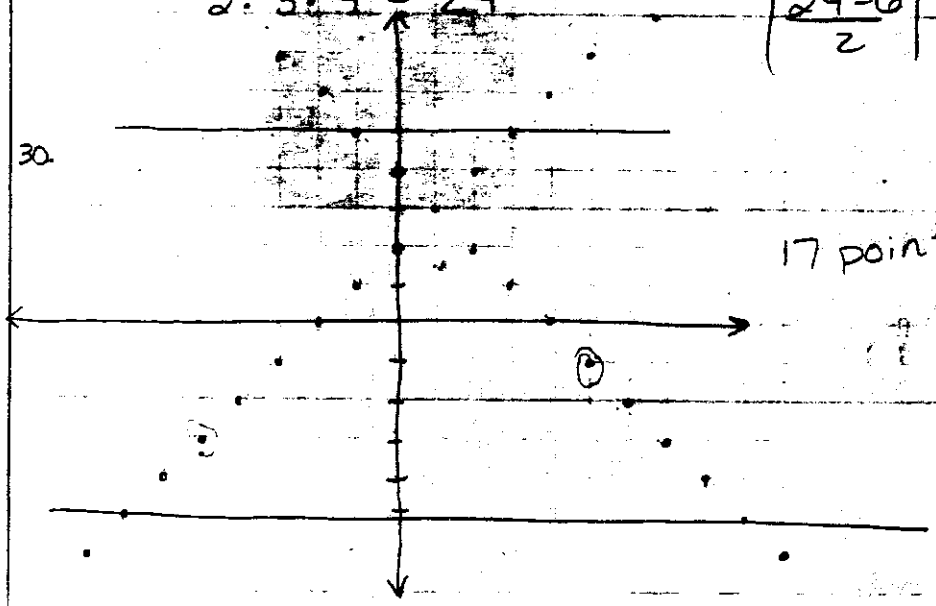
K even $K(K-1)(K+1)$

K must be multiple of 2
one of these is a multiple of 3

$2 \cdot 3 = 6$

$\left|\frac{24-6}{2}\right| = \left|\frac{18}{2}\right| = 9$ **D**

30.



17 points **B**

TB1 57069 → DEED

TB2 $19 \equiv 5 \bmod 7$

$19^2 \equiv 25 \bmod 7 \equiv 4 \bmod 7$

$19^3 \equiv -1 \bmod 7$

$19^6 \equiv 1 \bmod 7$

$19^{18} \equiv 1 \bmod 7$

$19^{19} \equiv 5 \bmod 7$

$99 \equiv 1 \bmod 7$

$99^{99} \equiv 1 \bmod 7$

remainder $\boxed{6}$

TB3 792 divides $13xy45z$ mult. of 11, 9, 8

ends in 452 454 $\boxed{456}$ 458 450

$13xy456$ $1+3+x+y+4+5+6 = 19+x+y = 27$ or 36

so $x+y = 8$ or $x+y = 17$

$1+x+4+6-(3+y+5) = 3+x-y = 0$ or 11

$x-y = -3$ or $x-y = 8$

$x+y=8$	$x+y=17$	$x+y=8$	$x+y=17$
$x-y=8$	$x-y=8$	$x-y=-3$	$x-y=-3$
$(8,0)$	$2x=25$	$2x=5$	$2x=14$
	$\varnothing$	$\varnothing$	$x=7$
			$y=10$ $\varnothing$

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