# Mu Alpha Theta National Convention: Seattle, 1997

## Theta Number Theory Topic Test

1. How many factors does 8120 have?

   (A) 32 (B) 28 (C) 24 (D) 20 (E) NOTA

2. In how many zeros does 500! end?

   (A) 500 (B) 124 (C) 432 (D) 218 (E) NOTA

3. What is the sum of the LCM and GCF of 48, 64, and 160?

   (A) 1042 (B) 824 (C) 976 (D) 888 (E) NOTA

4. What is the greatest prime less than 340?

   (A) 331 (B) 335 (C) 337 (D) 339 (E) NOTA

5. Determine the sum of the squares of all the natural numbers up to m.

   (A) $\frac{m(m+1)}{2}$ (B) $\frac{m(2m+1)}{2}$ (C) $\frac{2m(2m+1)}{3}$ (D) $\frac{m(m+1)(2m+1)}{6}$ (E) NOTA

6. Evaluate: $\sum_{i=0}^{12}\binom{12}{i}$

   (A) 3642 (B) 3880 (C) 4096 (D) 4234 (E) NOTA

7. How many prime numbers are factors of 60! ?

   (A) 21 (B) 11 (C) 23 (D) 17 (E) NOTA

8. There are 14 numbers less than n each of which is divisible by 2, 5, 6, 8, and 15. What values are possible for n?

   (A) $1681 < n < 1800$ (B) $n > 1800$ (C) $n > 1681$
   (D) $1801 < n < 1920$ (E) NOTA

9. For each n in the set of natural numbers from 1 to 8, the remainder when t is divided by n is 1. What is the smallest possible value for t?

   (A) 639 (B) 1121 (C) 841 (D) 4321 (E) NOTA

10. A two digit number is 45 greater than the number formed when its digits are reversed. What is the largest possible number that fits this description?

(A) 82 (B) 92 (C) 94 (D) 81 (E) NOTA

11. Two positive numbers, a and b, are both cubes and differ by 999. What is the second smallest possible value of a?

(A) 512 (B) 2197 (C) 729 (D) 1331 (E) NOTA

12. $A = \sum_{q=1}^{32} q!$ What is the remainder when A is divided by 14?

(A) 7 (B) 1 (C) 9 (D) 5 (E) NOTA

13. The number 5A427B is divisible by 6. How many possible values are there for A+B?

(A) 2 (B) 3 (C) 4 (D) 6 (E) NOTA

14. Evaluate: $3+\cfrac{3}{3+\cfrac{3}{3+\ldots}}$

(A) 4 (B) 5 (C) $\frac{3+\sqrt{21}}{2}$ (D) $\frac{3+\sqrt{3}}{2}$ (E) NOTA

15. Evaluate: $\sqrt{5+\sqrt{5+\sqrt{5+\ldots}}}$

(A) $\frac{1+\sqrt{21}}{2}$ (B) $\frac{5}{2}$ (C) $5-\sqrt{5}$ (D) $5-\frac{\sqrt{5}}{2}$ (E) NOTA

16. Express the base 10 number 5742 in base 17.
Note: Letters represent digits greater than 9, in alphabetic order. I.e. A=10, B=11, etc.

(A) 145A (B) 12ED (C) G94 (D) 13D9 (E) NOTA

17. What is the product of $234_5$ and $432_5$, expressed in base 5?

(A) $212342_5$ (B) $142412_5$ (C) $244143_5$ (D) $223231_5$ (E) NOTA

18. What is the sum of the positive factors of 2187?

(A) 3280 (B) 3192 (C) 3145 (D) 3473 (E) NOTA

19. What is the product of the positive factors of 512?

(A) $2^{52}$ (B) $2^{45}$ (C) $2^{43}$ (D) $2^{42}$ (E) NOTA

20. Which of the following numbers is both triangular and perfect?

(A) 21 (B) 28 (C) 36 (D) 45 (E) NOTA

21. A store sold 72 pencils for \$A84.B4. Determine A-B, if a pencil costs an integer number of cents.

(A) 1 (B) 5 (C) -1 (D) -5 (E) NOTA

22. How many Pythagorean triangles have side lengths which are consecutive natural numbers?

(A) 0 (B) 1 (C) 2 (D) infinitely many (E) NOTA

23. What is the units digit of $7^{171}$?

(A) 7 (B) 9 (C) 3 (D) 1 (E) NOTA

24. What are the last two digits of $11^{111}$?

(A) 71 (B) 51 (C) 91 (D) 11 (E) NOTA

25. $d = 4 \bmod 9$. Which of the following could d equal in mod 27?

(A) 23 (B) 18 (C) 13 (D) 8 (E) NOTA

26. A number m, when written in base c (c>7), is equal to AB, where A is c-4 and B is 7. What is m, expressed in base (c-2)?

(A) $247_{c-2}$ (B) $124_{c-2}$ (C) $103_{c-2}$ (D) $47_{c-2}$ (E) NOTA

27. If $x = a \bmod 17$, where a is a natural number, is a solution of the equation $6x = 7 \bmod 17$, determine the sum of the possible values of a.

(A) 4 (B) 7 (C) 10 (D) 15 (E) NOTA

28. After performing an ordinary arithmetic multiplication, a woman replaced all the even digits with the letter E, and all the odd digits with the letter O, to yield:

$$\begin{array}{cccc} & O & E & E \\ & & E & E \\ \hline E & O & E & E \\ E & O & E & \\ \hline O & O & E & E \end{array}$$

What is the product in the above problem?

(A) 9744 (B) 5382 (C) 7168 (D) 5546 (E) NOTA

29. In how many zeros does 48! end when expressed in base 6?

(A) 20 (B) 22 (C) 24 (D) 25 (E) NOTA

30. Find the sum of the three smallest natural numbers with exactly 14 factors.

(A) 818 (B) 841 (C) 882 (D) 960 (E) NOTA