

Mu Alpha Theta Convention 2004
Theta Matrices and Determinants Test

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 6$, then evaluate $\begin{vmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 2g & 2h & 2i \end{vmatrix}$.

- A. 6 B. 12 C. 24 D. 48 E. NOTA

2. Given: $X = \begin{bmatrix} 4 & 7 \\ 13 & 19 \end{bmatrix}$ and a = sum of the elements in row 1, b = product of the elements in column 2, and c = determinant of matrix X. Find: $\left(\frac{bc^2}{c^{-1}} - ac^3\right)c^{-4}$

- A. 7 B. $\frac{122}{15}$ C. -7 D. $\frac{-122}{15}$ E. NOTA

3. If A is a matrix and $|A^T| = 5$, then evaluate $|A^{-1}|$

- A. 5 B. -5 C. $\frac{1}{5}$ D. $\frac{-1}{5}$ E. NOTA

4. A is a square matrix, I is the identity matrix of the same order as A and $|A| = 3$. Which of the following is not necessarily equal to 3?

- A. $|AI|$ B. $|IA^T|$ C. $\frac{1}{|A^{-1}A^T|}$ D. $\frac{1}{|(A^{-1})^T|}$ E. NOTA

5. Find the sum of the entries of $\begin{bmatrix} 2 & 9 \\ 1 & 6 \end{bmatrix}^{-1}$

- A. 6 B. $\frac{4}{3}$ C. $\frac{-2}{3}$ D. $\frac{16}{9}$ E. NOTA

6. Given: $A = \begin{bmatrix} 5 & 18 \\ 3 & 22 \\ 6 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 7 & 3 \\ 4 & 2 & 5 \end{bmatrix}$. What are the dimensions of AB ?

- A. 2×3 B. 3×3 C. AB is not defined D. 2×2 E. NOTA

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7.
$$\begin{bmatrix} 1 & 2 & 3 \\ 5 & 2 & 0 \\ 9 & 6 & 4 \end{bmatrix} + \begin{bmatrix} \frac{3}{2} & 9.2 & 4 \\ 7 & 3 & 6 \\ 0 & 3 & 7 \end{bmatrix} = C$$
 What is the value of C_{32} ?

- A. 9 B. 6 C. 0 D. 18 E. NOTA

8. If matrix A , $A = \begin{bmatrix} 1 & 2 & a \\ 3 & 4 & b \\ 5 & 6 & c \end{bmatrix}$, has an inverse, then $|A^{-1}| = ?$

- A. $\frac{-0.5}{a-2b+c}$ B. $-2a+4b-2c$ C. abc D. 0 E. NOTA

9. Solve for x if $\begin{vmatrix} 2 & x \\ -4 & y \end{vmatrix} = 10$ and $\begin{vmatrix} 7 & -1 & x \\ 0 & 3 & y \\ 1 & 2 & -3 \end{vmatrix} = -57$

- A. -2 B. 2 C. -3 D. 3 E. NOTA

$$x + 2y + kz = 5$$

10. $9x - ky + z = 7$ Find the sum of all values of k such that the system has no solution.

$$kx + y + 7z = 15$$

- A. $\frac{-7}{8}$ B. 0 C. 21 D. 27 E. NOTA

11. $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} X + \begin{bmatrix} 3 & 8 \\ 7 & 17 \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ 5 & 7 \end{bmatrix}$. Solve for $|X|$ if X is a matrix.

- A. 228 B. 56 C. 0 D. -56 E. NOTA

12. Find $\sqrt{x+y}$, given $\begin{bmatrix} 2x & 3 \\ 4y & 7 \end{bmatrix} + \begin{bmatrix} 3y & 7 \\ 8x & 6 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 1 & 13 \end{bmatrix}$

- A. $\frac{7}{4}$ B. $\frac{49}{16}$ C. $\frac{\sqrt{7}}{2}$ D. $\frac{\sqrt{29}}{4}$ E. NOTA

13. Let $E = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad - cb = 4$. What is $fg - hg$ given $E^{-1} = \begin{bmatrix} f & g \\ h & j \end{bmatrix}$?

- A. 1 B. 4 C. $\frac{1}{4}$ D. 16 E. NOTA

14. Given: $A = \begin{bmatrix} 1 & 2 \\ c & 4 \end{bmatrix}$ and $|A| = 2$. $D = \begin{bmatrix} b & 6 \\ 7 & 8 \end{bmatrix}$ and $|D| = -26$. If $AD = C$, what is $|C|$?
- A. 1 B. 2 C. -21 D. -52 E. NOTA

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15. Given $Z = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \\ 5 & 9 & 13 \end{bmatrix}$. If $a =$ the number of rows in Z
 $b =$ the number of rows in Z^T
 $c =$ element Z_{23}
 $d =$ element Z_{12}
 $e = |Z|$, then find $e \begin{vmatrix} a & c \\ b & d \end{vmatrix}$.
- A. 6 B. 3 C. -6 D. -18 E. NOTA

16. Find the area of a triangle with vertices $(1, -1)$, $(3, 4)$ and $(0, 2)$.
- A. 5 B. 5.5 C. 6 D. 11 E. NOTA

17. Which of the following is/are not characteristics of matrices in row echelon form?
- I. All nonzero rows are preceded by zero rows (if both are present).
II. The last (right) nonzero element of each row is 1
III. When the first nonzero element of a row appears in column C, then all elements in column C in succeeding rows are zero
- A. I, II, III B. II, III C. II only D. III only E. NOTA

18. Given $A = \begin{bmatrix} 2 & 7 \\ -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 & -1 \\ 1 & 6 & -3 \end{bmatrix}$, which of the following are true?
- I. The determinant of A is positive.
II. The product AB yields a 2×3 matrix.
III. The product BA yields a 2×2 matrix.
IV. The determinant of the inverse of A equals the determinant of A .
- A. I only B. I, II and IV C. III only D. I and II E. NOTA

19. If the determinant of $\begin{vmatrix} x & y & 0 \\ 0 & 2 & -1 \\ -y & x & y \end{vmatrix} = 49$ and the determinant of $\begin{vmatrix} x & 6 \\ 4 & y \end{vmatrix} = -20$, find $x^3 + y^3$.
- A. ± 200 B. ± 343 C. ± 164 D. ± 259 E. NOTA

20. If $x^3 - 8x^2 + 17x = 10$, find the sum of all possible determinants of $\begin{vmatrix} x & -1 & x \\ 4 & 1 & 3 \\ 5 & 7 & 2 \end{vmatrix}$
- A. 10 B. 11 C. 12 D. 13 E. NOTA

21. Which matrix satisfies the equation: $3M^2 - 2M - \begin{bmatrix} 58 & 75 \\ 50 & 83 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

- A. $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ B. $\begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}$ C. $\begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$ D. $\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$ E. NOTA

22. There is a 50% chance of rain tomorrow if it is raining today. If it is sunny today, there is only a 30% chance of rain tomorrow. Given that it is sunny today, what is the chance of rain this time next week?

- A. 30% B. 37.5% C. 50% D. 62.5% E. NOTA

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23. Given the three points $(x, 2), (3, 4), (5, y)$ are collinear, find the value of $4x - xy + 3y$.

- A. -26 B. -16 C. 16 D. 26 E. NOTA

24. Which of the following matrices commute?

I. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

II. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

III. $\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$

- A. III only B. I and II C. I and III D. I, II and III E. NOTA

25. If X and Y are $n \times m$ matrices and Z is an $n \times k$ matrix, then which of the following are not true?

- A. $(X + Y)^T = X^T + Y^T$ B. $(XZ)^T = Z^T X^T$ C. $(X^T)^T = X$

- D. $|X^T| = |X|$ E. NOTA

26. If $A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & x^2 & 3 \\ 2 & 5 & 2 \end{bmatrix}$ and the determinant of A is 17, solve for x .

- A. -1 B. 1 C. 3 D. 4 E. NOTA

27. Given $A = \begin{bmatrix} -2 & -9 \\ 1 & 4 \end{bmatrix}$, $A^n =$

- A. $\begin{bmatrix} 1-3n & -9n \\ n & 1+3n \end{bmatrix}$ B. $\begin{bmatrix} -3n & -9n \\ n & 3n \end{bmatrix}$ C. $\begin{bmatrix} 1+3n & 1-9n \\ -n & 1-3n \end{bmatrix}$ D. $\begin{bmatrix} 3n & 1-3n \\ n & 1+3n \end{bmatrix}$ E. NOTA

28. Determine the cofactor of the element in row 2, column 3 of the matrix

$$\begin{bmatrix} 1 & 3 & 2 & 5 \\ -1 & 0 & -1 & 4 \\ -2 & -3 & 4 & 8 \\ 0 & 2 & 0 & 6 \end{bmatrix}$$

- A. 0 B. 18 C. -18 D. -12 E. NOTA

29. What is the element in the third row, first column of the adjoint of the matrix $\begin{bmatrix} 3 & 2 & 4 \\ 0 & 5 & 1 \\ -2 & 5 & -3 \end{bmatrix}$?

- A. -2 B. 4 C. 10 D. -18 E. NOTA

30. What is the sum of the eigenvalues of the matrix $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$?

- A. 2 B. 3 C. 9 D. 12 E. NOTA

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Tiebreakers

1. What values of x will satisfy the inequality $\begin{vmatrix} 5 & 2x \\ 3x+2 & -4 \end{vmatrix} > -20$?

2. Joel was shown the solution of x and y for a system of equations using Cramer's Rule. From this information, he was able to set up the solution for z . What value did he get for z ?

$$x = \frac{\begin{vmatrix} 1 & -3 & -1 \\ 3 & 1 & 1 \\ 0 & 2 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & -3 & -1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix}}, y = \frac{\begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 1 & 0 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & -3 & -1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix}}$$

3. What is the determinant of A^{-1} if $A = \begin{bmatrix} 1 & -3 & 1 & 0 \\ 0 & 3 & 1 & -1 \\ 5 & 2 & 0 & 1 \\ 1 & -3 & 1 & x \end{bmatrix}$?