

Mu Alpha Theta National Convention 2004
 Theta Matrices & Determinants
 Answers

#	Answer	#	Answer
1	D	18	D
2	D	19	D
3	C	20	B
4	C	21	A
5	C	22	B
6	B	23	C
7	A	24	Thrown out
8	A	25	Thrown out
9	D	26	E
10	B	27	A
11	E	28	B
12	D	29	C
13	C	30	A
14	D	TB1	$-2/3 < x < 0$
15	E	TB2	-2
16	B	TB3	$8-30x$
17	E		

Solutions:

1. D $8(6) = 48$

2. D $a = 11, b = 133, c = -15$
 $\left(\frac{bc^2}{c^{-1}} - ac^3\right)c^{-4} = \frac{b-a}{c} = \frac{133-11}{-15} = \frac{122}{-15}$

3. C $|A^T| = |A| = 5; |A^{-1}| = \frac{1}{|A|} = \frac{1}{5}$

4. C

5. C $\begin{bmatrix} 2 & 9 \\ 1 & 6 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ $2 - 3 - \frac{1}{3} + \frac{2}{3} = \frac{-2}{3}$

6. B $A = 3 \times 2$ $B = 2 \times 3$ $AB = 3 \times 3$

7. A $6 + 3 = 9$

8. A $|A| = \begin{vmatrix} 1 & 2 & a \\ 3 & 4 & b \\ 5 & 6 & c \end{vmatrix} = 4c + 10b + 18a - 20a - 6b - 6c$
 $-2a + 4b - 2c$ $|A^{-1}| = \frac{1}{-2(a - 2b + c)}$

$\begin{vmatrix} 2 & x \\ -4 & y \end{vmatrix} = 10$ $2y + 4x = 10$

9. D $\begin{vmatrix} 7 & -1 & x \\ 0 & 3 & y \\ 1 & 2 & -3 \end{vmatrix} = -57$ $-3x - 15y = 6$

Solving the system, $x = 3, y = -1$

10. B $\begin{vmatrix} 1 & 2 & k \\ 9 & -k & 1 \\ k & 1 & 7 \end{vmatrix} = 0$ $k^3 + 4k - 127 = 0$
sum of the roots of k are $\frac{-b}{a} = 0$

$$11. E \quad X = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 4 & 8 \\ -2 & -10 \end{bmatrix} = \begin{bmatrix} -1.6 & -5.6 \\ 2.8 & 6.8 \end{bmatrix}$$

$$|X| = 4.8$$

$$12. D \quad 2x + 3y = 7 \quad 8x + 4y = 1$$

$$x = \frac{-25}{16}, \quad y = \frac{27}{8} \quad \sqrt{\frac{-25}{16} + \frac{27}{8}} = \sqrt{\frac{29}{16}} = \frac{\sqrt{29}}{4}$$

$$13. C \quad ab - cb = |E^{-1}| = \frac{1}{|E|} = \frac{1}{4}$$

$$14. D \quad 4 - 2c = 2 \quad 8b - 42 = -26$$

$$c = 1 \quad b = 2$$

$$\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 16 & 22 \\ 30 & 38 \end{bmatrix}$$

$$\begin{vmatrix} 16 & 22 \\ 30 & 38 \end{vmatrix} = -52$$

$$15. E \quad e = 0 \text{ so the answer} = 0$$

$$16. B \quad \begin{matrix} 1 & -1 & & .5(4 + 6 + 0 + 3 - 0 - 2) \\ 3 & 4 & & .5(11) \\ 0 & 2 & & 5.5 \\ 1 & -1 & & \end{matrix}$$

$$17. E$$

$$18. D \quad |A| = 13 \quad AB \text{ is a } 2 \times 3$$

$$BA \text{ is undefined} \quad |A^{-1}| = \frac{1}{13}$$

I and II are true

$$19. D \quad xy - 24 = -20 \quad x^2 + 2xy + y^2 = 49$$

$$xy = 4 \quad (x + y)^2 = 49$$

$$x + y = \pm 7$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 + y^3 = (\pm 7)(49 - 2(4) - 4) = \pm 259$$

$$20. B \quad 2x - 15 \quad 28x - 5x - 21x + 18$$

$$4x - 7$$

$$x^3 - 8x^2 + 17x - 10 = 0 \quad x = 1, 2, 5$$

$$4(1) - 7 = -3; \quad 4(2) - 7 = 1;$$

$$4(5) - 7 = 13 \quad -3 + 1 + 13 = 11$$

$$21. A \quad \text{use a calculator}$$

$$22. \text{ B} \quad \text{Let } M = \begin{array}{cc|c} & s & r \\ \hline .70 & .30 & s \\ .50 & .50 & r \end{array} \quad M^7 = \begin{array}{cc|c} & s & r \\ \hline .625 & .375 & s \\ .625 & .375 & r \end{array}$$

$$23. \text{ C} \quad 3 \text{ points are collinear if } \begin{vmatrix} x & 2 & 1 \\ 3 & 4 & 1 \\ 5 & y & 1 \end{vmatrix} = 0$$

$$4x + 10 + 3y - 20 - xy - 6 = 0$$

$$4x - xy + 3y = 16$$

24. thrown out

25. E all are true (question thrown out)

$$26. \text{ E} \quad 2x^2 + 18 + 40 - 4x^2 - 15 - 24 = 17$$

$$-2x^2 = -2 \quad x = \pm 1$$

$$\begin{bmatrix} -2 & -9 \\ 1 & 4 \end{bmatrix}^2 = \begin{bmatrix} -5 & -18 \\ 2 & 7 \end{bmatrix}$$

$$27. \text{ A} \quad \begin{bmatrix} -2 & -9 \\ 1 & 4 \end{bmatrix}^3 = \begin{bmatrix} -8 & -27 \\ 3 & 10 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -9 \\ 1 & 4 \end{bmatrix}^n = \begin{bmatrix} 1-3n & -9n \\ n & 3n+1 \end{bmatrix}$$

$$28. \text{ B} \quad -1 \begin{bmatrix} 1 & 3 & 5 \\ -2 & -3 & 8 \\ 0 & 2 & 6 \end{bmatrix} = -(-18) = 18$$

$$29. \text{ C} \quad +1 \begin{vmatrix} 0 & 5 \\ -2 & 5 \end{vmatrix} = 10$$

30. A sum of the eigenvalues = sum of the elements in the main diagonal
 $0 + 2 = 2$

$$\text{TB 1} \quad \frac{-2}{3} < x < 0$$

$$-20 - 2x(3x + 2) > -20$$

$$-2x(3x + 2) > 0$$

$$\frac{-2}{3} < x < 0$$

$$\text{TB 2 } -2 \quad Z = \frac{\begin{vmatrix} 1 & -3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & -3 & -1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix}} = \frac{-12}{6} = -2$$

$$\text{TB 3 } 8 - 30x$$

$$3 \begin{vmatrix} 0 & 1 & -1 \\ 5 & 0 & 1 \\ 1 & 1 & x \end{vmatrix} + 1 \begin{vmatrix} 0 & 3 & -1 \\ 5 & 2 & 1 \\ 1 & -3 & x \end{vmatrix}$$

$$3(0 + 1 - 5 - 0 - 0 - 5x) + 1(0 + 3 + 15 + 2 - 15x)$$

$$3(-4 - 5x) + (20 - 15x)$$

$$8 - 30x$$