5

1999 Mu Alpha Theta National Convention
Individual
Theta

1. How many integral values satisfy the following inequality: $-19 \leqslant x \leqslant 99$?

a. 19 b. 99 c. 118 d. infinitely many e. not given

2. If $-19x + 19 < 99$, what is the least integer in the solution set?

a. -5 b. $-4\frac{4}{19}$ c. $-4\frac{3}{19}$ d. -4 e. not given

3. Let A represent the base 10 number equivalent to 3717 base eight. What is the sum of the digits of A?

a. 28 b. 25 c. 18 d. 15 e. not given

4. As a simple fraction, what is the probability that a number selected from the set { 19, 20, 21, 22, ... , 97, 98, 99} will be a prime number?

a. $\frac{1}{9}$ b. $\frac{2}{9}$ c. $\frac{1}{3}$ d. $\frac{4}{9}$ e. not given

5. Compute the value of x if $\frac{1}{4} \bullet \frac{2}{6} \bullet \frac{3}{8} \bullet \frac{4}{10} \bullet \frac{5}{12} \bullet \ldots \frac{30}{62} \bullet \frac{31}{64} = 2^x$.

a. -48 b. -36 c. -24 d. -12 e. not given

6. Triangle ABC have the coordinates A(-5, 2), B(-2, 3) and C(-4, 7). Reflect A, B and C across the y-axis. Label the reflection of A, A'; B, B' and C, C' respectively. What is the sum of the x-coordinates of A', B' and C' ?

a. -12 b. -11 c. 11 d. 12 e. not given

7 Find x so that $27^2 \bullet 6^6 = 18^x$

a. 6 b. 5 c. 4 d. 3 e. not given

8. Find the sum of a + b + c + d so that $ax^3 + (b-3)x^2 - cx + d = 4x^3 + 7x - 3$

a. 11 b. 7 c. 2 d. -3 e. not given

9. If $\frac{t^2-t}{t^2-2t-3} \bullet \frac{t^2+2t+1}{t^2+4t} \div \frac{P}{2t^2-32} = \frac{t-1}{t-3}$ and $P = at^2 + bt + c$ where $a > 0$ then what is $a + b + c$?

a. 14 b. 12 c. -6 d. -7 e. not given

10. The diagonals of a rhombus are 10 cm. and 24 cm. long. Find its perimeter, in cm.

a. 240 b. 120 c. 104 d. 52 e. not given

11. Find the equation of the perpendicular bisecting line of the segment PQ where P(6, 1) and Q(4, -9). Write your answer in $Ax + By = C$ form, where A, B and C are integers and $A > 0$. Find the value of $A + B + C$.

a. -21 b. -9 c. 9 d. 21 e. not given

12. Simplify: $(6-4)(4-6) \div (6+4)(4+6)$

a. -4 b. -0.04 c. 0.04 d. 4 e. not given

13. If the graphs of $2y + x + 3 = 0$ and $3y + ax + 2 = 0$ intersect forming right angles, then the value of a is

a. 6 b. 4 c. 2 d. -2 e. not given

14. Find the sum of all positive integers b which have the property that $\log_b 64$ is a postive integer.

a. 94 b. 78 c. 64 d. 14 e. not given

15. Evaluate: $6.\overline{3} - 2.1\overline{6}$

a. $4\frac{17}{100}$ b. $4\frac{7}{45}$ c. $4\frac{2}{15}$ d. $4\frac{1}{6}$ e. not given

16. If $f(x) = x^2 - 4$ and $g(x) = 2x - 1$, find the product of the solutions of $f(g(x)) - g(f(x)) = 12$

a. -12 b. -9 c. -6 d. -3 e. not given

17. Given Square PQRS and Equilateral Triangle, RST, where T lies inside Square PQRS. A segment from T to Q is drawn. What is the measure of Angle RQT? (Note: Label Square PQRS in a clockwise fashion starting in the upper left hand corner with P. All answers are in degrees.)

a. 68 b. 72 c. 75 d. 84 e. not given

18. Find the sum of all x-coordinates that satisfy the following system of equations:
$x^2 - 3y^2 = 13$ and $x = 1 - 3y$

a. 5 b. 3 c. -1 d. -9 e. not given

19. Circles O and P, with radii 20 and 34 units, respectively, intersect at points A and B. If AB is a common internal chord for both circles O and P and the length of AB is 32 units, then find the numerical length of line segment OP.

a. 54 b. 42 c. 32 d. 28 e. not given

20. Evaluate: $199919981997^2 - 2 \bullet 199919981994^2 + 199919981991^2$

a. 45 b. 36 c. 27 d. 18 e. not given

21. If acute angled triangle ABC whose measures are Angle A = $(x + 15)^\circ$, Angle B = $(2x - 6)^\circ$, and the exterior angle at C has measure $(3x + 9)^\circ$. Compute the number of possible integral values of x.

a. 18 b. 20 c. 22 d. 24 e. not given

22. Given:
$A + B = 1$
$B + C = 2$
$C + D = 3$
$D + E = 5$
$E + F = 8$
$F + G = 13$
$G + H = 21$
$H + I = 34$
$I + J = 55$
Determine the numerical value of A + J.

a. -45 b. -21 c. 34 d. 87 e. not given

23. Find the remainder when $x^{16} + 5$ is divided by x + 1.

a. 6 b. 5 c. 4 d. 3 e. not given

24. Find the value, in simplest form, of the fraction: $\frac{2^{n-4}-2^{n+1}}{2^{n+4}}$

a. $\frac{9}{8}$ b. $\frac{7}{8}$ c. $\frac{3}{4}$ d. $\frac{5}{8}$ e. not given

25. From the group of Abe, Bill, Carl, Don, Ed, and Fred, three are to be chosen to go on blind dates. What is the probability that Abe and Bill will be chosen, but Carl won't be selected?

a. $\frac{1}{20}$ b. $\frac{3}{20}$ c. $\frac{7}{20}$ d. $\frac{11}{20}$ e. not given

26. The diameter of the base of a right circular cone is 18 cm and its altitude is 12 cm. Find its total surface area, in square cm.

a. 540π b. 459π c. 216π d. 189π e. not given

27. Evaluate: $\left(\log_x x^5 - \log_3 27\right) \div \left(\log_5 0.2 + \log_4 8\right)$

a. 4 b. 1 c. 0.25 d. -8 e. not given

28. The measures of the interior angles of a convex polygon are in an arithmetic progression. If the smallest angle is 100° and the largest angle is 140°, then the number of sides of this polygon is what?

a. 12 b. 10 c. 8 d. 6 e. not given

29. If A(3, 4) and C(7,10) are opposite vertices of rectangle ABCD and vertices B and D lie on the circle, $x^2 + y^2 - px - qy + s = 0$, then what is the numerical value of p + q + s.

a. 77 b. 81 c. 87 d. 91 e. not given

30. When multiplied out, 15! (15 factorial) is equal to 130767A368000. Compute the missing digit A.

a. 1 b. 4 c. 7 d. 0 e. not given

Theta Individual Tie Breakers.

T1. The height of a cone is equal to the radius of its base. The radius of the sphere is equal to the radius of the base of the cone. The ratio of the volume of the cone to the volume of the sphere is what?

T2. If $4^x = \sqrt{2^{3y}}$ then solve for x in terms of y.

T3. If two of the three roots of $x^3 + px + q = 0$ are 3 and -1, find the third root.