

Mu Alpha Theta National Convention 2004
Theta Geometry

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. The sum of the measures of the interior angles of a regular polygon is 2520° .
How many sides does the polygon have?

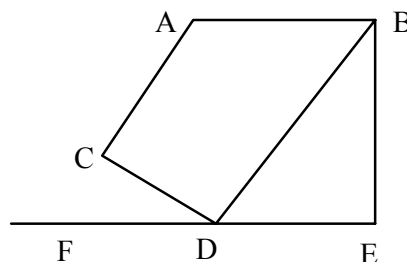
A) 7 B) 12 C) 14 D) 16 E) NOTA

2. In a rhombus, one angle is 120° , and the shorter diagonal is 12 inches.
Find the area of the rhombus.

A) $36\sqrt{3}$ square inches B) $72\sqrt{3}$ square inches C) 144 square inches
D) $144\sqrt{3}$ square inches E) NOTA

3. In the given figure $\overline{AB} \parallel \overline{DE}$, $\overline{AC} \parallel \overline{BD}$,
 $\overline{BE} \perp \overline{DE}$, $m\angle FDC = 30^\circ$, and $m\angle C = 85^\circ$.
Find $m\angle A$.

A) 95°
B) 115°
C) 125°
D) 135°
E) NOTA

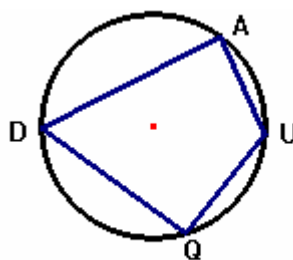


4. A, B, C, and D are distinct coplanar points, no three of which are collinear.
If E is a point not in the plane of A, B, C, and D, how many distinct planes
are determined by the five points?

A) 7 B) 6 C) 5 D) 4 E) NOTA

5. Quadrilateral QUAD is inscribed in a circle.
If $m\angle Q = 4x + 40$, $m\angle A = 5x + 5$, and
 $m\angle D = 3x + 10$, then find $m\angle U$.

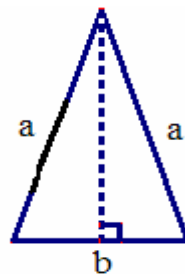
A) 15°
B) 55°
C) 125°
D) 135°
E) NOTA



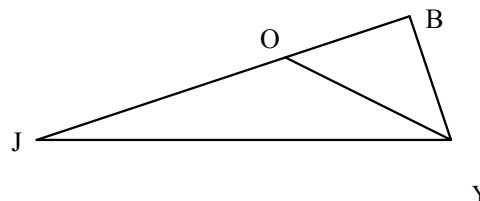
6. A cube has a volume of 60 m^3 . A second cube has sides whose lengths are
half that of the first cube. What is the volume of the second cube?

A) 7.5 m^3 B) 15 m^3 C) 30 m^3 D) 120 m^3 E) NOTA

7. Given the isosceles triangle pictured, $a > b$. Put the orthocenter, circumcenter, and centroid in order from top to bottom as they would appear on the dotted line.
- A) orthocenter, centroid, circumcenter
 B) centroid, orthocenter, circumcenter
 C) circumcenter, centroid, orthocenter
 D) centroid, circumcenter, orthocenter
 E) NOTA



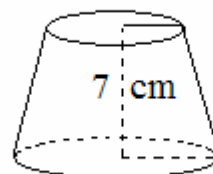
8. Find the area of triangle JOY given $JO = 8$ mm, $BY = 3\sqrt{2}$ mm, $m\angle JBY = 90$ and $m\angle BOY = 45$.
- A) $24\sqrt{2}$ mm²
 B) $12\sqrt{2} + 18$ mm²
 C) $12\sqrt{2} + 9$ mm²
 D) $12\sqrt{2}$ mm²
 E) NOTA



9. A diameter and a chord of a circle intersect in such a way that the diameter splits the chord into two segments of lengths 8 and 9 centimeters and the chord splits the diameter into two segments whose lengths are in the ratio 2:1. What is the measure of the diameter of the circle in centimeters?
- A) 6 B) 12 C) 17 D) 18 E) NOTA
10. A regular hexagon is inscribed in a circle. Another circle is inscribed inside the hexagon. What is the ratio of the area of the smaller circle to the area of the larger circle?
- A) $\frac{3}{4}$ B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$ E) NOTA

11. If the statements, “If X, then Y” and “If not Z, then not Y” are true, which of the following statements must also be true?
- A) If not Y, then not Z. B) If Y, then not X. C) If not Z, then not X.
 D) If not X, then not Z. E) NOTA

12. A frustum of a right circular cone has radii of upper base 4 cm and lower base 6 cm with height 7cm. What is the volume of the frustum?
- A) 364π cm³
 B) $\frac{532}{3} \pi$ cm³
 C) 175π cm³
 D) $\frac{140}{3} \pi$ cm³
 E) NOTA



13. A barn with a flat roof is rectangular in shape, 10 yards wide, 13 yards long, and 5 yards high. It is to be painted inside and outside, and on the ceiling,

but not on the roof or floor. What is the total number of square yards to be painted?

- A) 360 B) 460 C) 490 D) 590 E) NOTA

14. A regular hexagon is inscribed in a circle of radius 8 inches. What is the area enclosed by the hexagon?

- A) $32\sqrt{3} \text{ in}^2$ B) 48 in^2 C) 96 in^2 D) $96\sqrt{3} \text{ in}^2$ E) NOTA

15. Trapezoid ABCD, with $\overline{AB} \parallel \overline{CD}$, has median \overline{XY} , $AB = 12 \text{ cm}$ and $XY = 17 \text{ cm}$. Find CD.

- A) 14.5 cm B) 22 cm C) 24 cm D) 29 cm E) NOTA

16. Find the ratio of the area of a 45° - 45° - 90° triangle inscribed in a circle to the area of the part of the circle remaining when the triangle is removed?

- A) $1 : (\pi - 1)$ B) $1 : \sqrt{2} \pi$ C) $\pi : (\sqrt{2}\pi - 1)$
D) $1 : (\pi - \sqrt{2})$ E) NOTA

17. A clear plastic rectangular box is tightly sealed and contains a blue liquid.

The box has dimensions of 2 inches by 3 inches by 6 inches. When the box is sitting on its smallest face the liquid is 1 inch from the top. When the box is sitting on its largest face what is the distance (in inches) the liquid will be from the top?

- A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) 1 D) $\frac{5}{3}$ E) NOTA

18. A rectangular barn is 40 feet wide and 50 feet long. A horse is tied to a corner of the barn by a 50-foot long rope. How many square feet of ground outside of the barn can the horse graze?

- A) 1600π B) 1650π C) 1875π D) 1900π E) NOTA

19. Determine the volume of the solid formed by revolving the region in the xy-plane bounded by the triangle with vertices $(2, 0)$, $(10, 0)$, and $(5, 4)$ about the x-axis.

- A) 16π B) $\frac{128}{3}\pi$ C) $\frac{192}{3}\pi$ D) $\frac{256}{3}\pi$ E) NOTA

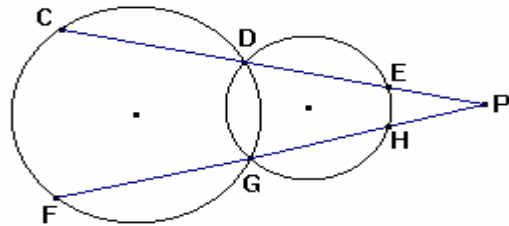
20. A square is inscribed in a circle. The region inside the square is painted blue and the remaining regions are painted red. If this is the layout for a dart board, what is the probability that a randomly thrown dart will land in one of the red areas?

- A) $\frac{1}{2\pi}$ B) $\frac{1}{5}$ C) $\frac{2}{\pi}$ D) $\frac{\pi-2}{\pi}$ E) NOTA

21. In the given figure, $PE = 9$, $DE = 3$, $DC = 6$, and $PH = 8$. Find FG .

- A) $\frac{16}{3}$
 B) $\frac{5}{2}$
 C) $\frac{11}{2}$
 D) $\frac{8}{3}$

E) NOTA



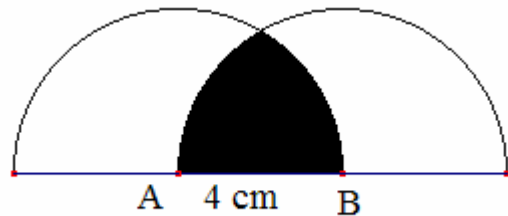
22. In a trapezoid the bases are 5 inches and 8 inches, and the legs are 4 inches and 6 inches. If the legs are extended to meet in a point, how much must the shorter leg be extended?

- A) 4 inches B) $5\frac{1}{3}$ inches C) $6\frac{2}{3}$ inches
 D) 10 inches E) NOTA

23. Suppose semicircles with centers A and B each have radius 4 cm. What is the area of the shaded portion?

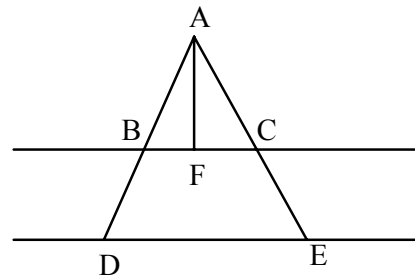
- A) $\frac{16\pi}{3}$ cm²
 B) $\left(\frac{16\pi}{3} - 4\sqrt{3}\right)$ cm²
 C) $\frac{8\pi}{3}$ cm²
 D) $\left(\frac{8\pi}{3} - 4\sqrt{3}\right)$ cm²

E) NOTA



24. Suppose the area of $\triangle ABC$ is 10 cm² and the area of $\triangle ADE$ is 50 cm². $\overline{BC} \parallel \overline{DE}$ and $\overline{AF} \perp \overline{BC}$. The distance between \overline{BC} and \overline{DE} is 5 cm. What is the length of \overline{AF} ?

- A) $1.25(5 + \sqrt{5})$ cm
 B) $1.25(\sqrt{5} + 1)$ cm
 C) $2.5(\sqrt{5} + 1)$ cm
 D) $2.5(\sqrt{5} - 1)$ cm



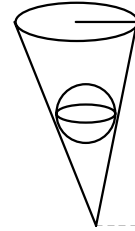
E) NOTA

25. A regular octagon is to be formed by cutting congruent isosceles right triangles from the corners of a square. If a side of the square has length 1, then what is the length of a leg of each of the triangles?

- A) $\frac{2+\sqrt{2}}{2}$ B) $\frac{2-\sqrt{2}}{3}$ C) $\frac{2-\sqrt{2}}{2}$
 D) $\frac{1+\sqrt{2}}{3}$ E) NOTA

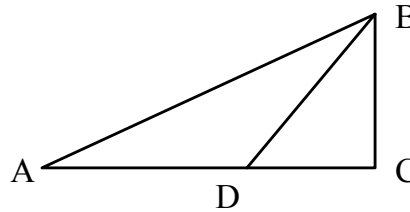
26. A sphere of radius 2 is placed inside a right circular cone of base radius 4 and height 10. What is the distance from the vertex of the cone to the center of the sphere?

- A) $\sqrt{29} - 2$
 B) 5
 C) $\sqrt{29}$
 D) $5\sqrt{2}$
 E) NOTA



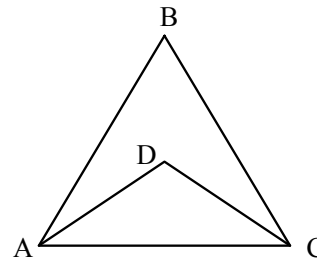
27. In $\triangle ABC$, $\overline{BC} \perp \overline{AC}$, $\angle A = 30^\circ$, $\angle BDC = 45^\circ$ and $AD = 12$. What is the length of \overline{BC} ?

- A) $\frac{6}{\sqrt{3}-1}$
 B) $\frac{12}{\sqrt{3}+1}$
 C) $6\sqrt{2} - 1$
 D) $6(\sqrt{3}+1)$
 E) NOTA



28. In $\triangle ABC$, $AB = BC = 5$. $AD = DC = 3$ and $AC = 4$. Find the area of quadrilateral ABCD.

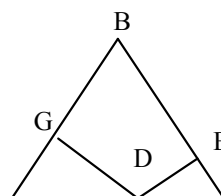
- A) $2\sqrt{21} - 2\sqrt{5}$
 B) $2\sqrt{21}$
 C) $2\sqrt{27} - 2\sqrt{5}$
 D) $2\sqrt{27}$
 E) NOTA



29. If the circumference of a circle is 100 cm, what is the measure of one side of a square inscribed in the circle?

- A) $\frac{25\sqrt{2}}{\pi}$ B) $\frac{50\sqrt{2}}{\pi}$ C) $\frac{100}{\pi}$ D) $\frac{100\sqrt{2}}{\pi}$ E) NOTA

30. If $\overline{DF} \perp \overline{AC}$, $\overline{DE} \perp \overline{BC}$, $\overline{DG} \perp \overline{AB}$, $DC = 6$, $EC = 4$,



FC = 5 and DG = 8, what is the length of the altitude of the equilateral $\triangle ABC$?

- A) 14
- B) $\sqrt{55}$
- C) $6 + 2\sqrt{5}$
- D) $8 + 2\sqrt{5} + \sqrt{11}$
- E) NOTA

Tiebreaker Questions:

TB1. The ends of a glass tube are circles with radii 7 centimeters and 5.25 centimeters. The segment joining the centers of the circles has length 6 centimeters and is perpendicular to each of the planes containing the circles. What is the outside surface area (in terms of π) of the tube in square centimeters, not including the ends?

TB2. Define a procedure P as follows:

P: Subdivide a square into 4 congruent squares and shade 3 of these congruent squares. Start with a square with side 256 inches and perform P on this square and then perform P on the new unshaded square. If we repeat this process on each new unshaded square until we have performed P on a total of 10 squares, what is the total area (in fraction form) of the shaded regions in square inches?

TB3. Suppose that $\angle ACD$ and $\angle DCB$ have the same measure, $AC = 18$, $BC = 12$, $AD = 9$, and $BD = 6$. What is the exact value of the length of \overline{CD} ?

