2000 Mu Alpha Theta National Convention

GEOMETRY SOLUTIONS

1. (C) Since the fold would bisect the radius that bisects the chord, the radius drawn to the endpoint of the chord would form the third side of a 30°–60°–90° triangle with the short side 3 inches, hypotenuse 6 inches, and the longest leg which is one-half the chord $3\sqrt{3}$ inches. The chord would then be twice that of $6\sqrt{3}$ inches.

2. (B) Let a represent the angles, then one-half a is the bisected angle and

$$\frac{1}{2}a = (90 - a) - 30$$

$$\frac{1}{2}a = 60 - a$$

$$\frac{3}{2}a = 60$$

$$a = 40$$

3. (D) Let x be the angle. Then we take one-half x, and one-half of one-half x, and finally one-half of that, i.e. one-eighth x.

$$\frac{1}{8}x + x = 180$$

$$\frac{9}{8}x = 180$$

$$x = 160$$

4. (C) By the Pythagorean theorem, the other leg of the large right triangle is 48. Therefore

$$\frac{x}{14} = \frac{48 - x}{50}$$

$$50x = 672 - 14x$$

$$64x = 672$$

$$x = \frac{21}{2}$$

Again using the Pythagorean theorem, for the smaller right triangle we get:

$$x^2 = \left(\frac{21}{2}\right)^2 + 14^2$$

$$x^2 = \frac{441}{4} + 196$$

$$x^2 = \frac{1225}{4}$$

$$x = \frac{35}{2}$$

5. (A) Heron's formula can be used to find the area of the triangle, 84 cm^2. Then the area formula using the area and the side of 14 cm can be used to find the altitude, $h = 12$ cm.

6. (B) The radii drawn to the points of tangency form a square with parts of the tangent segment. The extended piece makes the right triangle formed by the radius have a hypotenuse of 10 cm and one leg (the radius) of 8 cm, therefore the other leg is 6 cm. This makes one of the external segments 14 cm. The other radius is a segment parallel to one side of the other right triangle and this forms two similar triangles. Setting up the proportion $\frac{x}{x + 10} = \frac{8}{14}$, we find the smaller triangle side is $\frac{40}{3}$. Subtracting a radius, the extended length is $\frac{16}{3}$.

7. (A) Since the trapezoid circumscribes the circle the lengths of the two legs is the same as the sum of the bases (two tangents from the same external point are equal), and therefore each leg is 25 inches. An altitude from the short base vertex to the longer base makes a right triangle with hypotenuse equal to a leg, and short side found by subtracting the shorter base length from the longer base length and dividing by 2. Using the Pythagorean theorem, the altitude is 20 inches, the same as a diameter. Therefore the radius is 10 inches.

8. (B) Equation transforms to: $(x - 3)^2 + (y + 2)^2 = 64$, ∴ radius is 8.

9. (C) Since surface area equals volume, $6e^2 = e^3$, the edge is 6. The diagonal is $6\sqrt{3}$.

10. (B) Since the sum of the angles is $S = (n - 2)$ g 180, $n = 17$. The number of diagonals, $d = \frac{n(n-3)}{2}$ and $d = 119$.

11. (B) 1st, 2nd, 3rd, 4th, 7th, and 9th statements only, 6.

12. (E) Total number of possible common tangents exterior and interior is 10. Length or the median to the hypotenuse is one-half the hypotenuse, 5.

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13. (D) Drawing the segment from the exterior point to the center and from the center to a point of tangency will form a 30°–60°–90° triangle with short leg one-half the diameter, 4 cm. The length of each tangent segment is $\therefore\ 4\sqrt{3}$. The exterior segment of the secant times the whole secant equals the tangent segment squared:

$\left(8 - x\right) \times 8 = \left(4\sqrt{3}\right)^2$, x is the length of the chord of the secant. $x = 2$.

14. (A) The volume of the cylinder is 432π take away the volume of the sphere 288π, leaves 144π.

15. (C) First use the Pythagorean theorem to find the hypotenuse, 37. Then use the fact that the altitude forms similar triangles and geometric mean segments to the hypotenuse and pieces it is divided into to find the length of one of those segments. Then use either the Pythagorean theorem of those geometric mean segments to find the altitude. Alternate method, after finding length of hypotenuse, find area using legs and then find altitude using hypotenuse.

16. (C) The minor arc formed by the points of tangency of the two tangents from the same exterior point is always the supplement of the angle, therefore 140°.

17. (A) The midpoint of the diameter (center) is located at $\left(3, {}^{-}4\right)$. The length of a radius is $\sqrt{58}$. The equation

$$\left(x - 3\right)^2 + \left(y + 4\right)^2 = \left(\sqrt{58}\right)^2$$

simplifies to $x^2 + y^2 - 6x + 8y - 33 = 0$.

18. (E) Only III is true.

19. (C) The chord is 30 cm found by using the Pythagorean theorem and doubling the answer. Since the products of the segments of two intersecting chords are equal the segments are bound to be 12 cm and 18 cm. The longer segment is 18.

20. (C) If an altitude is dropped from the end of the shorter given leg to the longer given leg a 30°–60°–90° triangle is formed where the shorter given leg of 8 inches becomes the hypotenuse. The shortest leg of the right triangle is the altitude of the triangle whose area we are seeking, and is 4 inches. Using the area formula for a triangle with base 20 inches and height 4 inches, the area is 40 square inches.

21. (B) First find the midpoint of one of the segments (lets use the shorter segment). It is 1 cm from the point of intersection. This is the same distance that the longer chord is from the center of the circle. Use the Pythagorean theorem with the 1 cm segment, half the longest chord, 8 cm, and find the radius to be $\sqrt{65}$ cm.

22. (A) Simply take the area of the original rectangle, $20 \times 16 = 320$ square inches, and subtract the area of the right triangle that was cut out, $\frac{1}{2} \times 14 \times 12 = 84$ square inches to find the area of the pentagon, 236 square inches.

23. (B) The altitudes of the triangles are the same, the bases are the same, therefore the areas will be the same. The height of the original triangle is found using the Pythagorean theorem an is 9 cm. $\frac{1}{2} \times 24 \times 9 = 108$ square cm.

24. (C) Longest segment is opposite the largest angle, therefore longest segment is $\overline{OR}$.

25. (A) Area is one-half the apothem times the perimeter. Since the apothem is the altitude of one of the six congruent equilateral triangles forming the hexagon it is easy to find one side of the hexagon, $12\sqrt{3}$ cm, and the area, $648\sqrt{3}$ square cm.

26. (C) The area can be found by finding the areas of the four right triangles or more simply by using the fact that when the diagonals of a quadrilateral are perpendicular, the area is one-half the product of the diagonals. 285 square cm.

27. (A) In an isosceles trapezoid where the shorter base equals the length of the legs, a diagonal bisects the base angle formed with the longer base. Therefore $m\angle \text{PRA} = 120°$.

28. (C) The given arcs added is the arc intercepted by the angle opposite $\angle$ HMA and twice its measure. Opposite angles of an inscribed quadrilateral are supplementary. Therefore $m\angle \text{HMA} = 80°$.

29. (C) Using the sine function set up the equation $\sin 50° = \frac{\textit{opposite side}}{\textit{hypotenuse}}$. Solving we find the hypotenuse is approximately 50 mm.

30. (C) Only the centroid and the incenter must be concurrent in the interior of the triangle.