

Mu Alpha Theta National Convention 2004
Theta Gemini Test

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. Give the equation of the line which passes through the center of the circle $x^2 + y^2 - 14x + 10y + 73 = 0$ and is parallel to the line $x + 2y = 5$.

- A. $x + 2y = -4$
- B. $x + 2y = -3$
- C. $x + 2y = -1$
- D. $x + 2y = 7$
- E. NOTA

2. Simplify: $\frac{(3+i)^2}{(2-i)^4}$ where $i = \sqrt{-1}$.

- A. $\frac{8}{25} - \frac{6i}{25}$
- B. $\frac{-8}{25} + \frac{6i}{25}$
- C. $\frac{-80}{625} - \frac{234i}{625}$
- D. $\frac{80}{625} - \frac{234i}{625}$
- E. NOTA

3. Let $X = \{1, 2, 3, 4\}$. How many subsets of X do NOT contain the number 2?

- A. 6
- B. 7
- C. 8
- D. 9
- E. NOTA

4. Express this sum as a single number in base 8:
 $10102_3 + 1234_5 + 66_7$

- A. 504_8
- B. 511_8
- C. 516_8
- D. 523_8
- E. NOTA

5. Find the sum of the first 3 terms in the expansion of $(3 + 0.02)^7$.

- A. 2291.1012
- B. 2291.1141
- C. 2291.1240
- D. 2275.9380
- E. NOTA

6. Solve $\frac{e^x}{e^x - 2} = 3$ for x .

- A. $\frac{2}{3}$
- B. $\frac{3}{2}$
- C. $\ln \frac{3}{2}$
- D. $\ln 3$
- E. NOTA

7. A mixture of vinegar and water is 20% vinegar. To form a new mixture that is 30% vinegar, 40 quarts of the original mixture are removed and replaced with pure vinegar. How many quarts were there in the original mixture?

- A. 60
- B. 100
- C. 160
- D. 320
- E. NOTA

8. Solve for x : $\frac{x+2}{3} + \frac{y+1}{2} = 5$; $\frac{x+4}{4} - \frac{2y-5}{5} = 1$.

- A. 3
- B. 4
- C. 5
- D. 6
- E. NOTA

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9. The first three terms of a geometric sequence are $x - 1, x, x - 1$. Find the sum of these terms.

- A. $-\frac{1}{2}$
- B. $\frac{1}{2}$
- C. 1
- D. cannot be determined
- E. NOTA

10. Solve for x :

$$(A^{2x}B^{-3x})^{-4}(A^{-5}B^{-2x})^6 = 1, A \neq 1, B \neq 1, AB \neq 0.$$

- A. $-\frac{15}{8}$
- B. $-\frac{15}{4}$
- C. $\frac{15}{7}$
- D. $\frac{15}{2}$
- E. NOTA

11. $\{a_n\}$ is a geometric sequence,
 $a_4 = 54, a_7 = -16$. Find a_9 .

- A. $-\frac{64}{9}$
- B. $-\frac{16}{9}$
- C. $-\frac{2}{3}$
- D. $\frac{4}{9}$
- E. NOTA

12. Determine $\sum_{N=2}^7 \left(\frac{N}{N-1} - \frac{N+1}{N} \right)$.

- A. $-\frac{2}{7}$
- B. $\frac{1}{7}$
- C. $\frac{5}{7}$
- D. $\frac{6}{7}$
- E. NOTA

13. A right triangle has leg lengths 10 and 15. The largest possible square is cut from the triangle. Find the area outside the square and inside the triangle.

- A. 12
- B. 39
- C. 42
- D. 56
- E. NOTA

14. Find the coordinates of the point of intersection of the axes of symmetry for the graphs of the equations $y = x^2 - 3x + 2$ and $x = 2y^2 - 2y + 7$.

- A. (2,0)
- B. (2,7)
- C. $\left(\frac{3}{2}, \frac{1}{2}\right)$
- D. $\left(\frac{3}{2}, \frac{13}{2}\right)$
- E. NOTA

15. Find the sum of the real roots of
 $x - 3 = \sqrt{2x - 6}$.

- A. 2
- B. 3
- C. 5
- D. 8
- E. NOTA

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16. A rectangular solid container 7 inches by 18 inches by 3 feet is filled with water. All of the water is to be poured into cubical boxes with 6 inch edges. How many cubical boxes are needed to hold all of the water in the rectangular solid?

- A. 2
- B. 6
- C. 21
- D. 30
- E. NOTA

17. Let A = the slope of $3x + 2y = 9$,
B = the y-intercept of $5x - 3y = 9$,
C = the x-intercept of $4x - y = -10$.

Find the value of $\frac{4A+B}{C}$.

- A. $-\frac{9}{5}$
- B. $\frac{9}{5}$
- C. $\frac{38}{15}$
- D. $\frac{35}{12}$
- E. NOTA

18. Find the area of a trapezoid with bases 6 and 13 and legs 5 and $4\sqrt{2}$.

- A. $\frac{57}{2}$
- B. 38
- C. $\frac{95}{2}$
- D. 57
- E. NOTA

19. Solve for x:

$$12_3 + 12_5 + 12_7 + 12_9 + 12_x = 101110_2$$

- A. 10
- B. 11
- C. 12
- D. 13
- E. NOTA

20. What is the radius of the circle that passes through the points (5,3), (-2,2), and (-1,-5)?

- A. 5
- B. $4\sqrt{2}$
- C. 6
- D. $4\sqrt{3}$
- E. NOTA

21. Triangular numbers are determined by the formula $t = \frac{n(n+1)}{2}$ where n is a natural number.

What is the value of n that determines the triangular number 2701?

- A. 51
- B. 52
- C. 72
- D. 73
- E. NOTA

22. Simplify completely:

$$8 + \frac{1}{8 + \frac{1}{8 + \frac{1}{8 + \dots}}} - 2\sqrt{4 + \sqrt{4 + \sqrt{4 + \dots}}}$$

- A. 3
- B. 7
- C. $2 - \sqrt{17}$
- D. $\frac{7 - \sqrt{17}}{2}$
- E. NOTA

23. If the perimeter of an isosceles triangle is 40, and the altitude to the base is 8, find the length of the altitude to one of the legs.

- A. 12.8
- B. $12\frac{3}{11}$
- C. $11\frac{17}{29}$
- D. $10\frac{9}{11}$
- E. NOTA

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24. What is the mean of the terms in the following sequence: 17, 32, 47, ..., 302, 317 ?

- A. 143
- B. 167
- C. 173
- D. 201
- E. NOTA

25. Given rectangle ABCD with AB=DC=5 and BC=AD=3. Diagonal AC is divided into 3 congruent segments by points E and F. Find the area of triangle BEF.

- A. 5/2
- B. 15/4
- C. 5
- D. 15/2
- E. NOTA

26. Find the conjugate of the complex number $3i^{14} - 5i^{23}$. $i = \sqrt{-1}$.

- A. $-3 + 5i$
- B. $3 - 5i$
- C. $-3 - 5i$
- D. $3 + 5i$
- E. NOTA

27. The line which passes through the point (3, -3) with slope $-\frac{8}{3}$ also passes through the point (-6, k). Find the value of k.

- A. $\frac{3}{8}$
- B. $\frac{51}{8}$
- C. 16
- D. 21
- E. NOTA

28. A circular sheet of paper of radius 6 inches is cut into 3 equal sectors, and each sector is formed into a cone with no overlap. What is the height in inches of each cone?

- A. 4
- B. $3\sqrt{2}$
- C. $4\sqrt{2}$
- D. 6
- E. NOTA

29. Find the middle term of $\left(\frac{\sqrt{x}}{2} - x\right)^8$.

- A. $\frac{16}{3}x^4$
- B. $-\frac{25}{8}x^5$
- C. $-\frac{32}{5}x^5\sqrt{x}$
- D. $\frac{35}{8}x^6$
- E. NOTA

30. Find the axis of symmetry for the parabola

$$y = \frac{2}{3}x^2 + 3x + 5$$

- A. $x = -\frac{9}{2}$
- B. $x = -\frac{9}{4}$
- C. $x = -\frac{3}{2}$
- D. $x = -\frac{2}{3}$
- E. NOTA

Tiebreakers

1. In two similar rectangles, the ratio of the perimeters is 3:5. If the area of the smaller rectangle is 81, what is the area of the larger rectangle?

2. How many terms are contained in the sequence 5, 9, 13, 17, ..., 601?

3. If $x @ y = \frac{x}{y} + xy$, express $\frac{3}{8} @ \frac{3}{4}$ as a fraction in simplest form where the greatest common factor of the numerator and denominator is 1.