

Mu Alpha Theta National Convention 2004

Theta Gemini Test

For all questions, answer E. "NOTA" means none of the above answers is correct.

Mu Alpha Theta National Convention 2004

Theta Gemini

Answers

#	Answer	#	Answer
1	B	18	B
2	B	19	C
3	C	20	A
4	C	21	D
5	A	22	A
6	D	23	C
7	D	24	B
8	B	25	A
9	A	26	C
10	B	27	D
11	A	28	C
12	D	29	D
13	B	30	B
14	C	TB1	225
15	D	TB2	150
16	C	TB3	<u>25</u>
			32
17	E		

Mu Alpha Theta National Convention 2004
Theta Gemini Test

For all questions, answer E. "NOTA" means none of the above answers is correct.

Mu Alpha Theta National Convention 2004
Theta Gemini Test Solutions

1. B Center of the circle is (7,-5). $x + 2y = -3$

2. B $(3+i)^2 = 8+6i; ((2-i)^2)^2 = -7-24i;$
 $\frac{8+6i}{-7-24i} \cdot \frac{-7+24i}{-7+24i} = \frac{-8}{25} + \frac{6i}{25}$

3. C # of subsets is $2^n = 16$. Those with 2 are $\{2\}, \{1,2\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\},$
 $\{1,2,3,4\}, \{ \}$ which is 8. $16-8=8$

4. C Converting all to base 10: $10102_3 = 92, 1234_5 = 194, 66_7 = 48.$
 Sum is 334 which is 516 base 8.

5. A $3^7 + \frac{7!}{6!}(3^6)(.02) + \frac{7!}{5!2!}(3^5)(.02)^2;$
 $2187+7(729)(.02)+(21)(243)(.0004)=2291.1012$

6. D $e^x = 3e^x - 6; 6 = 2e^x; 3 = e^x; x = \ln 3$

7. D Let $x =$ the original mixture $0.2(x - 40) + 40 = .3x, x = 320$

8. B $2x + 4 + 3y + 3 = 30$ which simplifies to $2x + 3y = 23; 5x + 20 - 8y + 20 = 20$
 simplifies to $5x - 8y = -20$. Solving the system, $x=4$.

9. A $\frac{x}{x-1} = \frac{x-1}{x}; x = \frac{1}{2};$ terms are $-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2};$
 Sum is $-\frac{1}{2}$

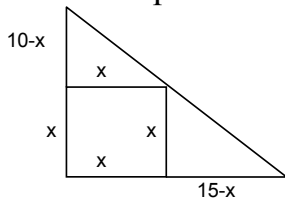
10. B $\left(\frac{B^{3x}}{A^{2x}}\right)^4 \left(\frac{1}{A^5 B^{2x}}\right)^6 = 1; \frac{B^{12x}}{A^{8x}} \cdot \frac{1}{A^{30} B^{12x}} = 1; \frac{1}{A^{8x} \cdot A^{30}} = 1;$
 $\frac{1}{A^{8x+30}} = 1; A^{8x+30} = A^0; 8x+30 = 0; x = -\frac{15}{4}$

11. A Let a_4 be the 1st term: $54, 54x, 54x^2,$
 $54x^3 = -16, 54x^4, 54x^5$ Since $54x^3 = -16, x = -\frac{2}{3}, a_9 = 54 \cdot -\frac{2^5}{3} = -\frac{64}{9}$

12. D Simplifying $\frac{N^2 - (N^2 - 1)}{N(N-1)} = \frac{1}{N^2 - N}; \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} = \frac{6}{7}$

Mu Alpha Theta National Convention 2004
Theta Gemini Test

For all questions, answer E. "NOTA" means none of the above answers is correct.



13. B

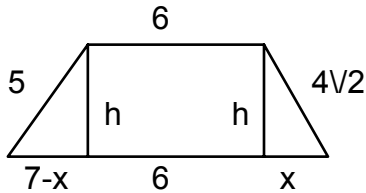
The two smaller triangles are similar by AA. Therefore, the sides are proportional. $\frac{10-x}{x} = \frac{x}{15-x}$.
Solving gives $x=6$. The area of the triangle is 75, the area of the square is 36. The difference is 39.

14. C $y = (x^2 - 3x + \frac{9}{4}) + 2 - \frac{9}{4}; y = (x - \frac{3}{2})^2 - \frac{1}{4}; x = \frac{3}{2};$
 $x = 2(y^2 - y + \frac{1}{4}) + 7 - \frac{1}{4}; x = 2(y - \frac{1}{2})^2 + 6\frac{3}{4}; y = \frac{1}{2} \quad (\frac{3}{2}, \frac{1}{2})$

15. D Squaring both sides, $x^2 - 6x + 9 = 2x - 6$.
Solving gives $x=5, x=3$ sum = 8.

16. C Using all units as inches: $\frac{7 \cdot 18 \cdot 36}{6^3} = 21$

17. E. $A = -\frac{3}{2}, B = -3, C = -\frac{5}{2} = \frac{18}{5}$



18. B

The Pythagorean Theorem can be used in both triangles. Then solve each equation for h^2 and set the equal. $25 - (7-x)^2 = 32 - x^2$; Solving gives $x = 4$ which is the height.
 $A = \text{median} \cdot \text{height} = \frac{19}{2} \cdot 4 = 38$.

Mu Alpha Theta National Convention 2004
Gemini Theta Solutions

19. C $12_3 = 5, 12_5 = 7, 12_7 = 9, 12_9 = 11,$
 $101110_2 = 46$
 $5 + 7 + 9 + 11 + 12_x = 46; 12_x = 14, x = 12$

20. A Let (x,y) be the center of the circle. Using the points and equation of a circle, solve this system. $(x-5)^2 + (y-3)^2 = (x+2)^2 + (y-2)^2$;

Mu Alpha Theta National Convention 2004

Theta Gemini Test

For all questions, answer E. "NOTA" means none of the above answers is correct.

$$(x+2)^2 + (y-2)^2 = (x+1)^2 + (y+5)^2.$$

Expanding and simplifying gives the system $\begin{cases} 14x + 2y = 26 \\ 2x - 14y = 18 \end{cases}$. The solution is (2,-1) which is the center. Using any point on the circle and the center, radius = 5.

21. D $73 \cdot 2701 = \frac{n^2 + n}{2}; n = 73$

22. A Let $y = 8 + \frac{1}{8 + \frac{1}{8 + \frac{1}{8 + \dots}}}$, substituting gives $y = 8 + \frac{1}{y}; y = y = 8 + \frac{1}{y}; y = 4 + \sqrt{17}$.

Let $x = \sqrt{4 + \sqrt{4 + \sqrt{4 + \dots}}}$, squaring both sides gives $x^2 = 4 + x \cdot x = \frac{1 + \sqrt{17}}{2}$. So

$$4 + \sqrt{17} - 2\left(\frac{1 + \sqrt{17}}{2}\right) = 3$$

23. C Let x be the congruent sides of the triangle. That makes the base $40 - 2x$. If you draw the altitude to the base, this makes a right triangle with hypotenuse x and legs 8 and $20 - x$. Using the Pythagorean theorem, $x = 11.6$. The area of the triangle is $\frac{1}{2} \cdot 8 \cdot 11.6 = 67.2$. Using the area and one of the legs as the base,

$$67.2 = \frac{1}{2}(11.6)(h). \quad h = 11\frac{17}{29}.$$

24. B Find the number of terms. $317 = 17 + 15(n - 1); n = 21$. Find the sum of the terms: $\frac{21}{2}(17 + 317), = 3507$.

Find the average by dividing the sum by the number of terms which is 167.

25. A The area of triangle ABC = $\frac{1}{2} \cdot 5 \cdot 3 = \frac{15}{2}$. Triangles AEB, EFB, and FCG have equal areas since they have congruent bases and congruent heights. Each triangle has an area that is $\frac{1}{3}$ of the area of triangle ABC.

$$\text{Area} = \frac{5}{2}.$$

26. C $3i^{14} = -3, 5i^{23} = -5i; -3 - (-5i)$ is $-3 + 5i$, conjugate is $-3 - 5i$

27. D Equation of line is $8x + 3y = 15$. Substituting the point $(-6, k)$ gives $-48 + 3y = 15. y = 21$

28. C Each sector has an area of 12π . Form the sector into a cone with lateral area 12π . Using the formula for lateral area with slant height of 6: $12\pi = \pi r l; 12\pi = \pi r \cdot 6, r = 2$. Using Pythagorean Theorem to find the height $6^2 = 2^2 + h^2, h = 4\sqrt{2}$.

Mu Alpha Theta National Convention 2004
Theta Gemini Test

For all questions, answer E. "NOTA" means none of the above answers is correct.

29. D $\frac{8!}{4!4!} \left(\frac{\sqrt{x}}{2}\right)^4 (-x)^4 = \frac{35}{8} x^6$

30. B $y = \frac{2}{3} \left(x^2 + \frac{9}{2}x + \frac{81}{16}\right) + 5 - \frac{27}{8}; x = -\frac{9}{4}$

TB 1. Answer: 225

Ratio of perimeters is 3:5. Ratio of areas is 9:25. $\frac{9}{25} = \frac{81}{x}, x = 225$

TB 2. Answer: 150

$$601 = 5 + 4(n - 1), n = 150$$

TB 3. Answer: $\frac{25}{32}$

$$\frac{3}{8} \div \frac{3}{4} + \frac{3}{8} \cdot \frac{3}{4} = \frac{25}{32}$$