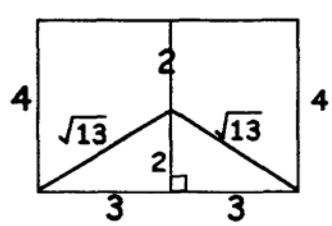
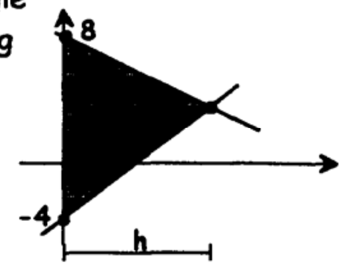
Solutions : Theta Gemini, Nationals 2000

1. C
2. $12-3x=x$ and $24+x=3y$ so $x=3,\ y=9$ which gives the sum of x and y is 12 = C.
3. For the region $(2+x)>0$ and $(3x-6)<0$ the expression is equivalent to $(2+x)+-(3x-6)=-2x+8$, choice B.
4. $\frac{w}{600}=0.3$ and $\frac{w}{600+w}<0.28$ so substituting W=180 from first equation, we get $\frac{180}{600+x}<0.28$. Solving this inequality we get that x must be 300/7 or approx. 42.85 and therefore we must lose 43 games. The answer is C.
5. $f(2)=5$ and $g(5)=25+15=40$ so $4k+1=40$ which gives $k=9.75$. Choice C.
6. $\frac{9+10}{10}=1rem9;\frac{21+9}{9}=3rem3;\frac{10+20}{10}3rem0$

   so the answer is A. Trying all possible combinations of two numbers for the numerator, we cannot get a remainder of 5.
7. The perimeter of the pentagon is $2(\sqrt{13})+8+6$ $=2\sqrt{13}+14$ which is choice B.

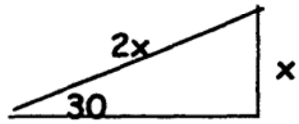


8. $f(3)=f(2)+6=1+6=7$, $f(4)=f(3)+6=7+6=13$, choice B.
9. Forty ounces of liquid were poured out, 30% of this was acid. So (40)(0.30) acid was poured out. So the Total acid (T) minus (40)(0.30) = 5. Solving $T-40(0.30)=5$ gives that the total must be 17. Choice A.
10. Subtracting the two equations gives $\log a-\log e=1$ so $\log(\frac{a}{e})=1$ so $10=a/e$ and $10e=a$ so if $a=10$ then e=1. Choice E.
11. The area of the triangle with base 12 units long is $\frac{1}{2}(12)h$ which is given to be 24. So h=4. The height of the triangle to base on the y-axis is the x-coordinate of the intersection of the lines. Setting the equations equal, $ax+8=bx-4$ gives $ax-bx=-12$ or $x(a-b)=-12$ or $x(b-a)=12$. Choice C.



12. The volume of the object is the volume of displacement, which is $\pi r^2(\text{change in height})$ which is $\pi(6^2)(\frac{1}{24})$ since half an inch is 1/24 of a foot. The volume is 1.5π or choice A.
13. If x>0 then $\sqrt{-2x}$ is imaginary and therefore $\sqrt{-2x}\cdot\sqrt{-2x}$ is equal to $i\sqrt{2x}\cdot i\sqrt{2x}$ which is negative. Since x>0, $-2x$ is also negative. If x<0, then $\sqrt{-2x}$ is positive and the product is positive. Since $x<0$, $-2x$ is also positive. Note that using absolute value would not give equivalent expressions for both x>0 and for x<0. The answer is choice D.
14. E. If the items of clothing started on the same day, then the brown shoes and blue jeans would coincide every blue day. But since they are one day off, we have a situation of 2x+1 and 4x+2 or an "even/odd" situation. They will never coincide, since the brown can be thought of as odd days and the blue as on even days. The brown and blue days will never coincide.
15. Using the change of base rule, $\frac{\log(x+4)}{2\log 2}=\frac{\log x}{\log 2}$ and multiplying by log(2) then simplifying gives $\frac{1}{2}\log(x+4)=\log x$. $\log\sqrt{x+4}=\log x$ so $\sqrt{x+4}=x$ and $x+4=x^2$ and $x^2-x=4$. Choice C.

16. $tr = d$ so distance is 2(150)=300 total distance. Distance = 2x + x = 3x. So $3x = 300$ and the vertical distance is x which is 100 feet. Choice A.



17. The numbers with the digit 5 are 5, 15, 25, 35, 45, 50 through 59, 65, 75, 85, 95. This totals 20 since 55 has two 5s. Choice B.

18. The log of the number is equal to $2000\log 2$ which approximates to 602.06 which indicates 603 digits. (Note the pattern if you start with the log of 2^5, which rounds up to 2 digits. The characteristic indicates number of digits in a logx number.) Choice D.

19. $20 = 2\pi r$ so $r = \frac{10}{\pi}$. Area of the circle is equal to $\pi\left(\frac{100}{\pi^2}\right)$ which simplifies to $\frac{100}{\pi}$. The difference in areas is $\frac{100}{\pi} - 25$ since a perimeter of 20 gives an area of the square is 25. Choice C.

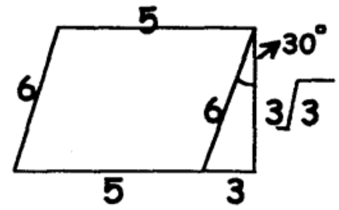
20. $2^2 = \log_3(\log_5 x))$ and $3^4 = \log_5 x$ so $5^{81} = x$. This number has 82 factors: which are $5^0, 5^1, 5^2, \ldots, 5^{81}$. Choice B.

21. $\frac{x+y}{2} = 3$ so $x + y = 6$ and squaring gives $x^2 + 2xy + y^2 = 36$. Since $\sqrt{xy} = 4$ from the geometric mean, $xy = 16$. Substituting, gives $x^2 + 2(16) + y^2 = 36$ so the sum of the squares of x and y is 36-32=4. Choice A.

22. The lateral surface plus the base area minus the drilled out base circles' area, plus the area of the inner lateral surface:
$2\pi(5)(20) + 2\pi(25) - 2\pi(2.5)^2 + 2\pi(2.5)(20)$
= 337.5π which is choice D.

23. $\frac{(x+3)(x+2)(x+1)!}{(x+1)!} = (x+3)(x+2) = 1003(1002)$ which is not divisible by 50. Choice B.

24. Area = bh = $5(3\sqrt{3})$ = $15\sqrt{3}$ or choice B.

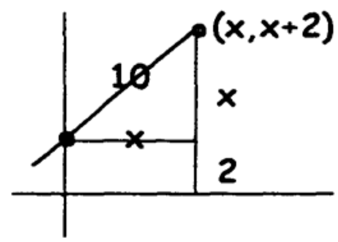


25. Choices: $\frac{1+4}{9} > 4$, $\frac{1+4}{9} > 9$, $\frac{4+9}{1} > 4$, $\frac{4+9}{1} > 9$, $\frac{9+1}{4} > 9$, $\frac{9+1}{4} > 1$. Only the third, fourth and last fit the criteria. The least value for the expression is therefore $\frac{9+1}{4} - 1 = 1.5$ which is choice B.

26. Factor out 9^{1996} to get $9^{1996}(9^3 - 9^2 - 9^1 + 1) = k \cdot 9^{1996}$ so k = 640, the quantity in the parentheses. Choice C.

27. Substituting gives $y = e^{\ln z}$ so $y = z$. Therefore $y = \frac{1}{m}$ and $m = \frac{1}{y}$. Choice C.

28. The distance from P to the point drawn is (5)(2 units/sec)=10 so the x-coordinate must be $5\sqrt{2}$ or choice E.



29. $(10 - 2x)(20 - 2x) = 40$
$4x^2 - 60x + 160 = 0$
so x is approximately 11.53 or 3.47.
Since 11.53 is impossible to the problem, 3.47 must be the answer. Choice C.

30. $A_1 = \frac{side^2}{4}\sqrt{3} = 36\sqrt{3}$ and $A_2 = 9\sqrt{3}$ since the side is 6. And so on. The sum is given by $\frac{a_1}{1-r}$ the sum of an infinite geometric sequence, or $\frac{36\sqrt{3}}{1-\frac{1}{4}} = 48\sqrt{3}$ or choice D.