2000 Mu Alpha Theta National Convention

"NOTA" in each question denotes "None of the Above"

1) Solve for x if $(x^2-4)(x+1)^2<0$

A) (-2,-1) B) (-2,2) C) $(-\infty,2)\cup(2,\infty)$ D) $(-2,-1)\cup(-1,2)$ E) NOTA

2) Given $f(x)=\frac{x-2}{x-4}$ and $g(x)=\frac{1}{x}$, what is the domain of $(f\circ g)(x)$?

A) $\{x : x\neq 0\}$ B) $\left\{x : x\neq 0, x\neq \frac{1}{4}\right\}$ C) $\{x : x\neq 0, x\neq 4\}$ D) $\left\{x : x\neq \frac{1}{4}\right\}$ E) NOTA

3) Which of the following functions represents the apothem (a) of a hexagon given its area (A)?

A) $a(A)=\sqrt[4]{\frac{A^2}{12}}$ B) $a(A)=\sqrt[4]{\frac{A^2\sqrt{3}}{12}}$ C) $A(a)=a^2\sqrt{3}$ D) $A(a)=2a^2$ E) NOTA

4) Find $f^3\left(f^{-1}(x)\right)+f(f(x))$ if $f(x)$ is defined by the following:

$$f(x)=\begin{cases}1 \text{ for rational numbers}\\ 0 \text{ for irrational numbers}\end{cases}$$

Let $f^2(x)$ denote $f(f(x))$

A) 1 B) $x+1$ C) $2x$ D) 2 E) NOTA

5) What is the domain of $g(x)=\sqrt{\log x}$?

A) $\Re$ B) $(0,\infty)$ C) $[0,\infty)$ D) $(1,\infty)$ E) NOTA

6) Given that $f(x)=x^3-8x^2+21x-20$ with roots r_1, r_2, and r_3, and that $g(x)=x^2-4$, what is the value of $g(r_1r_2+r_1r_3+r_2r_3)$?

A) -3 B) 60 C) 396 D) 437 E) NOTA

7) If $h(x) = ax^2 + bx + c$, where a, b, and c are natural numbers, which statement is NOT true?

A) the absolute extremum of h lies in either quadrant II or III
B) the zeros of h are non-negative
C) the graph of $|h(x)|$ contains the point $(0,c)$
D) the domain of h has no restrictions
E) NOTA

8) Let the only zeros of $f(x)$ be 3 and –4. What is the sum of the zeros of $g(x)$, given $g(x) = f\left(-\frac{x}{3}\right)$?

A) –3 B) $-\frac{1}{3}$ C) $\frac{1}{3}$ D) 3 E) NOTA

9) $f(x) = x^3 - (k-2)x^2 + 2x$ and f is an odd function. Find k.

A) –2 B) 0 C) 1 D) 2 E) NOTA

10) If the perimeter of the region bounded by $f(x) = |x-3| + 5$ and $g(x) = 7 - |x-2|$ is expressed in the form $a\sqrt{b}$, where a and b are relatively prime, find a^b.

A) 4 B) 8 C) 9 D) 16 E) NOTA

11) Given $f(x) = x^{2000} - x^{1999}$, what is the unit's digit of the remainder when $f(x)$ is divided by $(x-2)$?

A) 2 B) 4 C) 6 D) 8 E) NOTA

12) Find $f^{-1}(58)$ to the nearest thousandth if $f(x) = x^{4/3} - 8$.

A) 8.485 B) 22.627 C) 64.000 D) 216.511 E) NOTA

13) Given that $f(bx) = b \cdot f(x)$ for all real b, and $f(2) = 5$, find $f(20)$.

A) $\frac{1}{50}$ B) 17 C) 23 D) 50 E) NOTA

14) Which of the following are one-to-one functions?

I) $f(x) = e^x$ II) $f(x) = x^2$ III) $f(x) = 3x^7 - 4$

A) I and II only B) I and III only C) II and III only D) I, II, and III E) NOTA

15) A polynomial of odd degree with real coefficients can have which of the following?

A) all nonreal roots
B) an odd number of real roots
C) an odd number of nonreal roots
D) an even number of roots
E) NOTA

16) The recursive Fibonacci function can be defined as $f(n) = f(n-1) + f(n-2)$ for the nth term, where $f(1) = 1$ and $f(2) = 1$. Find $f(35)$.

A) 3,524,578 B) 5,702,887 C) 9,227,465 D) 14,930,352 E) NOTA

17) If $w(x) = \frac{ax+b}{cx+d}$, find $w^{-1}(x)$.

A) $\frac{cx-d}{ax-b}$ B) $\frac{dx-b}{a-cx}$ C) $\frac{x/a-b}{x/c-d}$ D) $\frac{x/c-b}{d-x/a}$ E) NOTA

18) Given function $h(n)$ on the positive integers, with $h(p) = \sqrt{p}$ for every prime p and $h(ab) = h(a) \cdot h(b)$, how many of the following are not necessarily true? (Note: k is an integer)

I) $h(1) = 1$
II) $h(p^k) = p^k$
III) $h(12)$ is real
IV) $h(a+b) = h(a) + h(b)$ for integers a and b

A) None B) One C) Two D) Three E) NOTA

19) The function $f(x, y, a) = x^2 - y^2 - a$ has zeros with $|y| \geq |x|$ if and only if

A) $a \in \Re$ B) $a < 0$ C) $a > 0$ D) $a = 0$ E) NOTA

20) Given that $f(1-x)+(1-x)f(x)=5$, find $f(5)$.

A) $-\frac{21}{20}$ B) $-\frac{20}{21}$ C) $\frac{20}{21}$ D) $\frac{21}{20}$ E) NOTA

21) The roots of $f(x)=x^3-\frac{1}{3}x^2-x-z$ form a harmonic progression when $\frac{1}{z}$ is:

A) -3 B) -1 C) 1 D) 3 E) NOTA

22) How many of the following functions are invertible?

$y_1=0$ $y_2=x^2$ $y_3=|x|$

A) All B) Two C) One D) None E) NOTA

23) Given that $f(x)=f(x-1)+1$, and $f(99)=100$, find $f(0)$.

A) -1 B) 0 C) 1 D) 2 E) NOTA

24) Let $r(n)$ be defined as the radius of the circle circumscribed about a regular n-gon of side length 1. For $r(x)=1, x=?$

A) 1 B) 3 C) 6 D) 12 E) NOTA

25) The function P is defined as such: $P(d)$ = probability that a fair die, tossed d times, lands with a prime number face up at least once. Find $P(4)$.

A) $\frac{1}{2}$ B) $\frac{3}{4}$ C) $\frac{7}{8}$ D) $\frac{15}{16}$ E) NOTA

26) If $f(x)=x^3-21x^2+84x-64$, then the product of all n such that $f(8^{n-1})=0$ is?

A) $-\frac{1}{4}$ B) $-\frac{1}{9}$ C) $\frac{1}{9}$ D) $\frac{1}{4}$ E) NOTA

27) If $f(x)=\sqrt{x+2}$ and $g(x)=x+1$, $f(g(x))=g(f(x))$ when $x=$

A) -2 B) -1 C) 1 D) 2 E) NOTA

28) $f(x) = 4x^2 - 9x + 1$ $g(x) = \sqrt{x}$ $h(x) = 4$
Find $h \circ g(x) + g \circ h(x) + f \circ g(9)$.

A) 12 B) 14 C) 16 D) 18 E) NOTA

29) Which of the following are either even or odd functions?

I) $y_I = -\frac{1}{x}$ II) $y_{II} = -x^4 + 1$ III) $y_{III} = [x - .5]$

A) I only B) II only C) III only D) I, II, and III E) NOTA

30) The height of a stable ice cream cone (in inches) can be represented by a function, inversely related to the thickness of the cone (in millimeters), to the number of scoops of ice cream, and to the square of the ice cream's temperature (in degrees Celsius). Thom's sugar cone has 1 scoop each of cookies n' cream, peanut butter fudge, and mint chocolate chip, and the 7" cone is 4 mm thick at a temperature of 30°C. If Thom gets a waffle cone that is 6.5mm thick and 16" tall, and he cools his ice cream to 10°C, what is the maximum number of whole scoops he can order while keeping the ice cream cone from becoming unstable?

A) 7 B) 8 C) 9 D) 10 E) NOTA