

1999 Mu Alpha Theta National Convention
Functions
Open Division

NOTA stands for None of these answers

Unless otherwise stated, the domain will be all real numbers which are meaningful to the problem.

1. Given $f(x) = 2x + 7$ and $g(x) = 3x + n$, find n so that $f(g(x)) = g(f(x))$

- A) 12 B) 13 C) 14 D) 15 E) NOTA

2. If $f(x) = 2^{x+1}$ and $g(x) = 3^x$ find $f(g(1))$

- A) 216 B) 36 C) 12 D) 16 E) NOTA

3. Suppose $f(x)$ is an even function and $g(x)$ is an odd function. What kind of function will $h(x) = f(x) \cdot g(x)$ be?

- A) even B) odd C) linear D) not enough information E) NOTA

4. Find the value of m and b so that $f(x) = \begin{cases} x^2 - 6x + 14 & \text{if } x > 3 \\ mx + b & \text{if } -3 \leq x \leq 3 \\ x^2 + 6x + 8 & \text{if } x < -3 \end{cases}$ is continuous.

- A) $m = \frac{-2}{3}$ $b = -3$ B) $m = \frac{2}{3}$ $b = 3$ C) $m = -1$ $b = 8$ D) $m = 1$ $b = 2$ E) NOTA

5. Suppose $f(x)$ is a polynomial with integer coefficients for which 3 and 13 are both roots. Which of the following could possibly be the value of $f(10)$?

- A) 3 B) 10 C) 14 D) 42 E) NOTA

6. Find all values of a such that the quadratic equation $x^2 + (a - 3)x + a = 0$ has two distinct positive real solutions.

- A) $a > 0$ B) $0 < a < 1$ C) $a < 1$ D) $a > 9$ E) NOTA

7. If $f(x)$ satisfies $2f(x) + f(1 - x) = x^2$ for all x , then $f(x) =$

- A) $\frac{x^2 - 3x + 1}{2}$ B) $\frac{x^2 + 8x - 3}{9}$ C) $\frac{4x^2 + 3x - 2}{6}$ D) $\frac{x^2 + 2x - 1}{3}$ E) NOTA

8. What is the domain of f if $f(x) = \frac{1}{|x| - 1}$?

- A) All real numbers B) All real numbers except 0 C) All positive real numbers
D) All non-negative real numbers E) NOTA

9. If $f(x) = |x - 1|$ and $g(x) = 1 - x^2$, then which of the following is $3f(-2) + 4g(-3)$?

- A) -23 B) -5 C) 13 D) 32 E) NOTA

10. Suppose that $f(n) = \log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdots \log_{(n-1)} n$. Then the value of $\sum_{k=2}^{10} f(2^k)$ is what?

- A) 48 B) 50 C) 52 D) 54 E) NOTA

11. Let $s(n)$ denote the sum of the digits on n . For example, $s(197) = 1 + 9 + 7 = 17$. Let $s^2(n) = s(s(n))$, $s^3(n) = s(s(s(n)))$, and so on. What is the value of $s^{1999}(1999)$?

- A) 28 B) 10 C) 1 D) can't be determined E) NOTA

12. Given that $f(x) = (x^5 - 1)(x^3 + 1)$, $g(x) = (x^2 - 1)(x^2 - x + 1)$, and $h(x)$ is a polynomial such that $f(x) = g(x)h(x)$, what is the value of $h(1)$?

- A) undefined B) 2 C) 3 D) 5 E) NOTA

13. Which of the following statements does NOT describe a function?

- A) Assign to each triangle its area B) Assign to each day its date C) Assign to each book its number of pages
D) Assign to each hour the temperature at that hour E) NOTA

14. Which of the following sets of ordered pairs is a function?

- A) $\{(x,y): y = x^2\}$ B) $\{(x,y): x = |y|\}$ C) $\{(x,y): y = \pm\sqrt{x}\}$ D) $\{(x,y): y < x\}$ E) NOTA

15. Which of the following statements describe the domain of the function defined by:

$$f(x) = \frac{\sqrt{x-3}}{(x-2)(x-\sqrt{3})}$$

- A) All real numbers except 2 and $\sqrt{3}$ B) All real numbers except those less than 3
C) All positive real numbers D) All positive real numbers except 2 and $\sqrt{3}$ E) NOTA

16. Which of the following terms describe(s) $f(x) = 0$?

- I The zero function II The constant function III The identity function

- A) I and III only B) II and III only C) I and II only D) I only E) NOTA

17. If $f(x) = 3x + 2$, what is the inverse of f ?

A) $g(x) = -3x - 2$

B) $g(x) = 3x - 2$

C) $g(x) = \frac{1}{3}x - \frac{2}{3}$

D) $g(x) = -\frac{1}{3}x - \frac{2}{3}$

E) NOTA

18. Select the TRUE statement about relations and functions from the following:

A) All relations are functions

B) The set of points on a circle represent a function

C) All functions are relations

D) A relation is a set of numbers

E) NOTA

19. The domain of a function is

A) the set of all points in the function

B) the value of x which makes the function zero

C) the same as the graph of the function

D) the set of all possible x values in the function

E) NOTA

20. Which of the following is a feature of the relation $|x| + |y| = 6$?

A) a sketch of the relation is a "diamond" shape with the x and y intercepts 6 units from the origin

B) The domain of the relation is $-3 < x < 3$

C) points satisfying the relation lie in the first and fourth quadrant only

D) the range of the relation is $0 < y < 6$

E) NOTA

21. Given $F(x) = 2x - 1$ and $G(x) = \frac{5}{x}$. If the inverse of $G [F(x)]$ is $K(x)$, the value of $K(4)$ is

A) $\frac{3}{2}$

B) $\frac{9}{8}$

C) $\frac{5}{7}$

D) $\frac{1}{8}$

E) NOTA

22. The composition of a function and its inverse

A) depends on the function

B) always gives the identity function $F(x) = x$

C) is the reflection of the function in the line $y = x$

D) depends on the order of the composition

E) NOTA

23. Find an expression for $f(4x)$ in terms of $f(x)$, given that $f(x) = \frac{x}{x-1}$.

A) $\frac{4f(x)}{3f(x) - 1}$

B) $\frac{4f(x)}{3f(x) + 1}$

C) $\frac{4f(x)}{4f(x) - 1}$

D) $\frac{4f(x)}{4f(x) + 1}$

E) NOTA

24. The roots of $f(x) = x^3 - 12x^2 + 37x - b$ form an arithmetic progression when b is

A) 20

B) 40

C) 60

D) 80

E) NOTA

25. Given the function $f(x)$ satisfying $f(x) + 2f\left(\frac{1}{1-x}\right) = x$, find $f(2)$.

- A) 5 B) $\frac{2}{3}$ C) $\frac{37}{8}$ D) $\frac{10}{7}$ E) NOTA

26. Which of the following statements is false?

- A) No cubic function can have a zero of multiplicity of four
B) A polynomial function of odd degree and having real coefficients must have at least one real zero
C) If the domain of " f " is (complex numbers) and if " f " is defined by a polynomial with real coefficients, then $a - bi$ is a zero of " f " if and only if $a + bi$ is a real zero of " f "
D) The product of the zeros of $f(x) = x^3 - x^2 - 12x$ is -12
E) NOTA

27. In interval notation, what is the range of the function $f(x) = \begin{cases} 2x + 3 & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$

- A) $(-\infty, 4)$ B) $(-\infty, 4]$ C) $(0, \infty)$ D) $[0, \infty)$ E) NOTA

28. The graph of $f(x) = \frac{x^2 - x - 2}{x + 2}$ has an oblique asymptote. The equation of this asymptote is:

- A) $y = 3 - x$ B) $y = x$ C) $y = x + 3$ D) $y = x - 3$ E) NOTA

29. Given $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x+3}$, where $f(x)$ and $g(x)$ are real-valued functions. Find the domain of $g(f(x))$.

- A) $\{x: x > 2\}$ B) $\{x: x \geq \frac{5}{3}\}$ C) $\{x: x < 2\}$ D) $\{x: x \leq \frac{5}{3}\}$ E) NOTA

30. If $f(x) = 1 + \frac{1}{x}$, find $f(f(f(x)))$

- A) $\frac{x+1}{x}$ B) $\frac{2x+1}{x+1}$ C) $\frac{3x+2}{2x+1}$ D) $\frac{4x+3}{3x+2}$ E) NOTA

Tiebreakers

T1. For what real number(s) m is the graph of $x^2 + y^2 - 4x - 6y = m - 13$ tangent to the graph of $x + y = 5 + \sqrt{2m}$

T2. Find the inverse of the function: $f(x) = \log_2 \frac{2x-1}{2}$.

T3. What is the period of the graph of $y = 4\cos^3 x \sin x - 4\cos x \sin^3 x$?