Functions Topic Test Key

1. B
2. C
3. C
4. D
5. C
6. C
7. A
8. B
9. E
10. B
11. D
12. E
13. D
14. A
15. B
16. C
17. E
18. C
19. D
20. C
21. C
22. C
23. C
24. B
25. B

Functions Topic Test Solutions

5

① $f(x) = x^3 + 2x^2 - 5$ ans B

$f(2) + f(-1) = 2^3 + 2^3 - 5 + (-1)^3 + 2 - 5$

$= 11 - 1 - 3$

$= 7$

② $f(x) = \log_b x$ ans C

$-\frac{3}{2} = \log_b \frac{1}{8}$

$b^{-\frac{3}{2}} = 2^{-3}$

$b^{\frac{1}{2}} = 2$

$b = 4$

③ $f(x) = \frac{x}{x+1}$ $(x, 3)$ ans C

$3 = \frac{x}{x+1}$

$3x + 3 = x$

$2x = -3$

$x = -\frac{3}{2}$

④ $f(x) = 4^x$ ans D

$f(x+2) - f(x+1) + f(x) = 4^{x+2} - 4^{x+1} + 4^x$

$= 16 \cdot 4^x - 4 \cdot 4^x + 4^x$

$= 13 \cdot 4^x$

⑤ ans [illegible]

$$\begin{cases} 3 = 16 + 4A + 2B - 5 \\ -37 = -16 + 4A - 2B - 5 \end{cases}$$

$-34 = 8A - 10$ $3 = 11 - 12 + 2B$

$8A = -24$ $2B = 4$

$A = -3$ $B = 2$

$A + B = -1$

⑥ $f(x+1) = x^2+3x+5$

$f(x) = ax^2+bx+c$

$x^2+3x+5 = a(x+1)^2+b(x+1)+c$

$x^2+3x+5 = ax^2+2ax+a+bx+b+c$ ans C

$a = 1$

$2+b = 3 \therefore b=1$

$a+b+c = 5$

$c = 3$

$\therefore f(x) = x^2+x+3$

alt. sol
decrease roots by 1

$$\begin{array}{r|rrr} -1 & 1 & 3 & 5 \\ & & -1 & -2 \\ \hline & 1 & 2 & 3 \\ & & -1 & \\ \hline & 1 & 1 & \end{array}$$

$f(x) = x^2+x+3$

⑦ $\frac{4}{2x-1} = 2\left(\frac{4}{x-1}\right)$ ans A

$\frac{1}{2x-1} = \frac{2}{x-1}$

$x-1 = 4x-2$

$-3x = -1$

$x = \frac{1}{3}$

⑧ $P(x) = (x+2)(x+k)$ ans B

$P(x) = x^2+(2+k)x+2k$

$$\begin{array}{r|rrr} 1 & 1 & 2+k & 2k \\ & & 1 & 3+k \\ \hline & 1 & 3+k & 3+3k \end{array}$$

$3+3k = 12$

$k = 3$

alt sol:

$12 = 3(1+k)$

$k = 3$

⑨ $y = x^2 - 3x + 2$ ans. E

$y = x^2 - 3x + \frac{9}{4} + 2 - \frac{9}{4}$

$y = x^2 - 3x + \frac{9}{4} - \frac{1}{4}$

$y + \frac{1}{4} = (x - \frac{3}{2})^2$

$\therefore y \geq -\frac{1}{4}$

⑩ $f(x) = \ln x$ $\quad g(8) = e^8$ ans B

$g(x) = e^x$

$f(e^8) = \ln e^8$

$f(e^8 = 8 \ln e$

$= 8$

⑪ Must be either C or D ans. D

(4 has to be a factor of last no.)

C) $(x^2-4)(x^2-1)$ No

$\therefore$ Must be D. Check.

2	1	−6	9	4	−12
		2	−8	2	+12
	1	−4	1	6	0
2		2	−4	−6	
	1	−2	−3	0	

yes

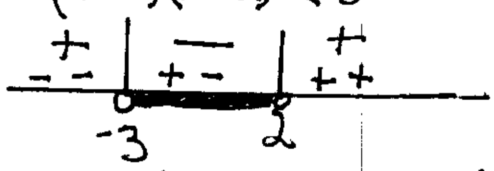
⑫ $D_{g \circ f}$. Take values in D_f ans. E

such that $f(x)$ is in D_g.

$\therefore x^2 + x - 1 < 5$

$x^2 + x - 6 < 0$

$(x+3)(x-2) < 0$



Take $(-3 < x < 2) \cap (x \geq -2)$

$-2 \leq x < 2$

⑬ $(h \circ g \circ f)x = h[g(f)](x)$

$$g[f(x)] = \frac{1}{cx+d}$$

$$h[g(f(x))] = \frac{a}{c} + \left(\frac{bc-ad}{c}\right)\frac{1}{cx+d}$$ ans D

$$= \frac{acx + ad + bc - ad}{c(cx+d)}$$

$$= \frac{\cancel{c}(ax+b)}{\cancel{c}(cx+d)}$$

⑭ $$y = \frac{4x^3 + 4x^2 - 3x - 3}{x^2 + 3x + 2}$$ ans A

$$y = \frac{4x^2(x+1) - 3(x+1)}{(x+1)(x+2)}$$

$$\begin{array}{r} 4x - 8 \\ x+2 \,\overline{)\, 4x^2 \quad 3} \\ \underline{4x^2 + 8x } \\ -8x \\ \underline{-8x - 16} \\ 19 \end{array}$$

$$y = \frac{\cancel{(x+1)}(4x^2-3)}{\cancel{(x+1)}(x+2)}$$

asymptotes $x = -2$ or $y = 4x - 8$

⑮ $$\lim_{x \to 2} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}$$ ans B

$$\lim_{x \to 2} \frac{2x+5-x-7}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{2x+5}+\sqrt{x+7})} = \frac{1}{3+3}$$

$$= \frac{1}{6}$$

(16) $f \circ g(x) = x \rightarrow$ f and g inverses

$\therefore$ Find $f^{-1}(x)$ which will be $g(x)$ ans. C

$$x = \frac{y+1}{y}$$

$$xy = y+1 \qquad \therefore g(x) = \frac{1}{x-1}$$

$$y = \frac{1}{x-1} \qquad g\left(\frac{x-1}{x}\right) = \frac{1}{\frac{x-1}{x}-1} = \frac{x}{-1}$$

(17)

$$\begin{array}{r|rrrr}
2 & 1 & -6 & 5 & 12 \\
 & & 2 & -8 & -6 \\
\hline
2 & 1 & -4 & -3 & 6 \\
 & & 2 & -4 & \\
\hline
2 & 1 & -2 & -7 & \\
 & & 2 & & \\
\hline
 & 1 & 0 & &
\end{array}$$

E

$$x^3 - 7x + 6 = 0$$

you could also use sum of roots:

$$r_1 + r_2 + r_3 = 6$$

If roots 2 less, sum will be 0.

(18) List possible fractions ans C

$\pm\frac{2}{3}$, $\pm\frac{1}{9}$, $-\frac{1}{3}$ D & E can't be.

$$\begin{array}{r|rrrrrr}
-\frac{1}{9} & 9 & 19 & 65 & 115 & 66 & 6 \\
 & & -1 & -2 & -7 & -12 & -6 \\
\hline
 & 9 & 18 & 63 & 108 & 54 & 0
\end{array}$$

(19) $16+8k_1=0 \longrightarrow k_1=-2$ ans D

$200-40-k_2=0 \longrightarrow k_2=160$

$r_1+r_1+10=2$

$r_1=-4$

$32+16-k_3=0 \longrightarrow k_3=48$

$k_1+k_2+k_3=206$

(20) A and B can be. ans C

Check C

$y=e^x \quad f^{-1}: x=e^y$

$\ln x \neq e^x \quad y=\ln x$

D and E both check

(21) $3r=-k$ ans C

$-\frac{k}{3}=r$

$-\frac{k^3}{27}+\frac{k^3}{9}+\frac{54k}{3}+216=0$

$k^3+243k+2916=0$

$$\begin{array}{r|rrrr} -9 & 1 & 0 & 243 & 2916 \\ & & -9 & 81 & -2916 \\ \hline & 1 & -9 & 324 & 0 \end{array}$$

$k=-9$

(22) $\tan 3x = \tan(2x+x)$

$$= \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

ans C

$$= \frac{\frac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \frac{2\tan^2 x}{1-\tan^2 x}}$$

$$= \frac{2\tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2\tan^2 x}$$

$$= \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

(23) $f(x+1) = \frac{2f(x)+1}{2}$; $f(1) = 2$

ans. C

$f(2) = \frac{2+1}{2} = \frac{3}{2}$

$f(3) = \frac{2\cdot\frac{3}{2}+1}{2} = 4$

$f(4) = \frac{2\cdot 4+1}{2} = \frac{9}{2}$

$f(37) = 2 + 36\cdot\frac{1}{2} = 20$

(24) $f(x) = \sqrt{3}\cos x + 3\sin x$

ans. B

$A = \sqrt{12} = 2\sqrt{3}$

$B = \dfrac{2\pi}{\frac{2\pi}{3}} = 3$

$AB = 6\sqrt{3}$

(25) $y = \log_e \dfrac{1+\sqrt{1-x^2}}{x}$

$e^y = \dfrac{1+\sqrt{1-x^2}}{x}$

ans. B

f^{-1}: $e^x = \dfrac{1+\sqrt{1-y^2}}{y}$

$e^x y = 1 + \sqrt{1-y^2}$

$e^x y - 1 = \sqrt{1-y^2}$

$e^{2x}y^2 - 2e^x y + \cancel{1} = \cancel{1} - y^2$

$(e^{2x}+1)y^2 - 2e^x y = 0$

$y[(e^{2x}+1)y - 2e^x] = 0$

$y \neq 0$ $\qquad y = \dfrac{2e^x}{e^{2x}+1}$

$y = \dfrac{2}{e^x + e^{-x}}$