

SOLUTION PAGE

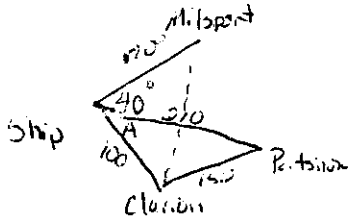
DIVISION: **THETA** - Nationals '96

TEST: **Equations and Inequalities**

4

<p>1. $\frac{ ax+by+c }{\sqrt{a^2+b^2}} \Rightarrow \frac{ 3(8)+(4)(2)+21 }{\sqrt{9+16}} = 10$ D.</p>	<p>7. $\begin{matrix} X & 60 & X+60 \\ 80 & 48 & 65 \\ 80x + 2880 = 65(x+60) \\ 80x + 2880 = 65x + 3900 \\ 15x = 1020 \\ x = 68 \end{matrix}$ C</p>
<p>2. $x^2+y^2-12x+16y-36=0$ $x^2-12x+36+y^2+16y+9=36+36+9$ $= 81$ $r=9$ $C=2\pi r$ $C=18\pi$ B</p>	<p>8. $\frac{1}{2}x + \frac{1}{5}x = 1$ $\frac{x}{10} + \frac{x}{20} = 1$ $\frac{2x+x}{20} = 1$ $3x=20$ $x=\frac{20}{3}$ D</p>
<p>3. slope of radius to (2,5) $\frac{5-3}{2-6} = -\frac{1}{2}$ slope of tangent line = 2 pt (2,5) slope 2 $\Rightarrow y-5=2(x-2)$ $2x-y=-1$ B</p>	<p>9. $\left(\frac{1}{4}\right)^{x+6} = 64^{-x}$ $\left(\frac{1}{4}\right)^{x+6} = \frac{1}{64^x}$ $\left(\frac{1}{4}\right)^{x+6} = \left(\frac{1}{4}\right)^{3x}$ C $x+6=3x$ $6=2x$ $3=x$</p>
<p>4. $7-3x < 2x+6$ $2x+6 < 8-4x$ $1 < 5x$ $6x < 2$ $\frac{1}{5} < x$ $x < \frac{1}{3}$ $\frac{1}{5} < x < \frac{1}{3}$ C</p>	<p>10. $J=6+3M$ $3J+4M=44$ $3(6+3M)+4M=44$ $13M=26$ $M=2$ $J=6+3(2)$ $J=12$ $J-M$ $12-2=10$ B</p>
<p>5. $x=1+\sqrt{5+x-1}$ $x-1=\sqrt{5+x-1}$ $x^2-2x+1=5+x-1$ $x^2-3x-3=0$ $x=\frac{3\pm\sqrt{9+12}}{2}$ $x=\frac{3\pm\sqrt{21}}{2}$ can't have neg answer so $\frac{3+\sqrt{21}}{2}$ D</p>	<p>20 = 8+y 12 = y 20 = $\frac{5}{4}x$ x=16 so AREA = $\frac{1}{2}(20)(16)$ $= 96$ D</p>
<p>6. $h=8+y \Rightarrow y=h-8$ $h=\frac{4}{3}x$ $x=\frac{3}{4}h$ $x^2+y^2=h^2$ $\left(\frac{3}{4}h\right)^2+(h-8)^2=h^2$ $h^2-25h+100=0$ $(h-5)(h-20)=0$ $h \neq 5$ so $h=20$</p>	<p>Page 1 of 4</p>

11.



$$120^2 = 100^2 + 100^2 - 2 \cdot 100 \cdot 100 \cos A$$

$$\frac{74}{105} = \cos A$$

$$A = 41.2^\circ$$

$$40 + 41.2 = 81.2$$

$$x^2 = 120^2 + 100^2 - 2 \cdot 120 \cdot 100 \cos(81.2)$$

$$x = 144 \text{ miles}$$

140 nearest ten miles

A

12. Cuttings	0	1	2	3	4	5	6
# of squares	1	4	10	14	22	34	49
perimeter of each	64	32	16	8	4	2	1

smallest square perimeter = $1 \times 4 = 4 \text{ cm}$

$$1076 \times 4 = 10384 \text{ cm}$$

$$\Rightarrow 103.84 \text{ m} \quad 104 \text{ m} \quad B$$

13. $\sqrt{12x} + 2\sqrt{18x} - 3\sqrt{9x} = -6\sqrt{4x} + 3\sqrt{8x} + \sqrt{9x} + \sqrt{32}$

$$2\sqrt{3x} = 4\sqrt{2}$$

$$12x = 32 \quad C$$

$$3x = 8$$

$$x = \frac{8}{3}$$

14. $\sqrt{2x+1} - \sqrt{x-1} = \sqrt{x+2}$

$$2x+1 = (\sqrt{x+2} + \sqrt{x-1})^2$$

$$2x+1 = x+2 + \sqrt{x^2+x-2} + x-1$$

$$2x+1 = 2x+1 + \sqrt{x^2+x-2}$$

$$0 = \sqrt{x^2+x-2}$$

$$0 = x^2+x-2$$

$$0 = (x+2)(x-1)$$

$$x = -2 \text{ or } x = 1$$

but -2 leaves - in $\sqrt{\quad}$
so $x = 1 \quad B$

15. $(3rs - 2t^2)^8$

$$\binom{8}{3} (3rs)^5 (-2t^2)^3$$

$$56 (243r^5s^5) (-8t^6)$$

$$-108,864r^5s^5t^6 \quad A$$

16.

1	.05
2	.10
3	.20
4	.40
5	.50
6	1.00

10. 25.60

15. 319.20

20. 26214.40

add them all
and
total
\$ 52,405.75
D

17. $r^2 = \sin^2\theta + \cos^2\theta$
 $x^2 + y^2 = 2 \frac{xy}{r} + \frac{x^2}{r^2} - \frac{y^2}{r^2}$
 $x^2 + y^2 = \frac{2xy + x^2 - y^2}{x^2 + y^2}$
 $(x^2 + y^2)^2 = x^2 + 2xy - y^2$
A

20. 3003_4
 $3(4^3) + 0(4^2) + 2(4^1) + 3(4^0)$
 $192 + 0 + 8 + 3 = 203_{10}$
 $6^2 \quad 6^1 \quad 6^0$
 $5(6^2) + 3(6^1) + 5(6^0) \Rightarrow 535_6$

18. $\frac{(n+3)!(n-1)!(n-2)!}{(n-2)!n!n!} = \frac{3}{8}$
 $\frac{(n+3)(n-1)}{n(n-1)(n-2)} = \frac{3}{8}$
 $\frac{n+3}{n(n-2)} = \frac{3}{8}$
 $8n+24 = 3n^2-6n$
 $0 = 3n^2-14n-24$
 $0 = (3n+4)(n-6)$
 $n=6$ or $n=-\frac{4}{3}$ but n is an integer
 so $n=6$ **E**

21. $\frac{6!}{3!2!} = \frac{6 \cdot 5 \cdot 4}{2}$
A
 $= 60$

22. $\log_3 9^{\frac{1}{5}} = \log_x 16^{\frac{1}{5}}$
 $\log_3 9^{\frac{1}{5}} \qquad \log_x 16^{\frac{1}{5}} = \frac{2}{5}$
 $3^n = 9^{\frac{1}{5}} \qquad x^{\frac{2}{5}} = 16^{\frac{1}{5}}$
 $3^n = 3^{\frac{2}{5}} \qquad x^{25} = 4^{25}$
 $n = \frac{2}{5} \qquad x = 4$
B

19. $\frac{C}{n \ 6} = 7 \binom{C}{n \ 7}$
 $\frac{n!}{6!(n-6)!} = 7 \frac{n!}{7!(n-7)!}$
 $\frac{(n-7)!}{(n-6)!} = \frac{6!}{6!}$
 $\frac{1}{n-6} = 1$
 $1 = n-6$
 $7 = n$
C

23. $\log_7(x^2+x) = \log_7 12$
 $x^2+x = 12$
 $x^2+x-12 = 0$
 $(x+4)(x-3) = 0$
 $x = -4$ or $x = 3$ **B**
 $(3, -4)$

