

Mu Alpha Theta National Convention: Seattle, 1997  
Theta Real and Complex Numbers Topic Test

1. Simplify:  $(2 - 3\sqrt{3}) - (4i - 5) + (-4 + i)$
- (A)  $3 - (3 + 3\sqrt{3})i$  (B)  $3 + (3\sqrt{3} - 3)i$  (C)  $(3 - 3\sqrt{3}) - 3i$  (D)  $(3 - \sqrt{3}) - \sqrt{3}i$  (E) NOTA
2. Evaluate:  $(1 - 2i\sqrt{7})(3\sqrt{7} + 4i)$
- (A)  $11\sqrt{7} - 38i$  (B)  $18\sqrt{7} - 46i$  (C)  $11\sqrt{7} + 46i$  (D)  $\sqrt{7}(8 + 23i)i$  (E) NOTA
3. Evaluate:  $(\sqrt{2} + i)^5$
- (A)  $9\sqrt{2} - 6i$  (B)  $8\sqrt{2} + 5i$  (C)  $-11\sqrt{2} + i$  (D)  $5\sqrt{2} - 3i$  (E) NOTA
4. Evaluate:  $\frac{6e^{2\pi i} \times 8e^{\frac{2\pi i}{3}}}{2e^{\frac{\pi}{2}}}$
- (A)  $8\sqrt{3} + 12i$  (B)  $12 + 16i$  (C)  $12 + 5i$  (D)  $12\sqrt{3} + 12i$  (E) NOTA
5. Which of the following are third roots of  $-i$ ?
- I.  $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$   
II.  $\frac{\sqrt{3}}{2} - \frac{1}{2}i$   
III.  $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$   
IV.  $i$
- (A) I & IV (B) I & II (C) III & IV (D) I, II, & IV (E) NOTA
6. What is the sum of the real roots of  $x^3 + 5x^2 - 11x + 21$ ?
- (A) 21 (B) -7 (C) -21 (D) -5 (E) NOTA
7. What is the magnitude of  $7 + 6i$ ?
- (A)  $\sqrt{13}$  (B)  $\sqrt{85}$  (C) 13 (D)  $2\sqrt{13}$  (E) NOTA

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8. Which of the following is equal to  $6e^{-\frac{9\pi i}{4}}$
- (A)  $-3\sqrt{2} - 3\sqrt{2}i$  (B)  $3\sqrt{2} - 3\sqrt{2}i$  (C)  $3\sqrt{2} + 3\sqrt{2}i$  (D)  $3\sqrt{2} - 3\sqrt{2}i$  (E) NOTA
9. Evaluate:  $\frac{4+2i}{3-2i}$
- (A)  $\frac{3+5i}{5}$  (B)  $\frac{16-14i}{5}$  (C)  $\frac{8+14i}{13}$  (D)  $\frac{6-2i}{13}$  (E) NOTA
10. Completely factor:  $x^3 + 5x^2 - 4x - 20$
- (A)  $(x^2 + 4)(x - 5)$  (B)  $(x + 1)(x - 2)(x + 10)$   
(C)  $(x + 2)(x - 2)(x + 5)$  (D)  $(x + 2)^2(x - 5)$  (E) NOTA
11. For what real values of  $x$  does  $2x^4 - 8x^2 = 90$ ?
- (A)  $x \in \{\pm 3\}$  (B)  $x \in \{\pm 3, \pm 1\}$  (C)  $x \in \{\pm 3, \pm\sqrt{5}\}$   
(D)  $x \in \{\pm 3, \pm i\sqrt{5}\}$  (E) NOTA
12. For what values of  $x$  does  $\frac{x+2}{x-2} + \frac{x-2}{x+2} = \frac{8-4x}{x^2-4}$ ?
- (A)  $x \in \{0\}$  (B)  $x \in \{-2, 3\}$  (C)  $x \in \{0, -2\}$  (D)  $x \in \{-1, 3\}$  (E) NOTA
13. Which of the following polynomials have roots of 2, -3, and 4?
- I.  $2x^3 + 6x^2 + 16 + 48$   
II.  $x^3 + 3x^2 + 8 + 24$   
III.  $x^3 - 3x^2 + -10 + 24$
- (A) III (B) I & II (C) I & III (D) II (E) NOTA
14. What is the remainder when  $3x^2 - 4x + 7$  is divided by  $x - 4$ ?
- (A) 25 (B) 28 (C) 39 (D) 17 (E) NOTA

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15. Which of the following may be a root of  $2x^3 - 4x^2 + kx - 6$ , if  $k$  is an integer?

- I.  $\frac{3}{4}$
- II.  $\frac{1}{2}$
- III.  $-3$
- IV.  $4$
- V.  $-\frac{3}{2}$

(A) II & IV      (B) I, III, & IV      (C) II, III, & V      (D) I, III, & V      (E) NOTA

16. What is the maximum value of  $f(x) = -2x^2 - 3x + 8$ ?

(A) 8      (B)  $\frac{65}{8}$       (C)  $\frac{67}{8}$       (D)  $\frac{73}{8}$       (E) NOTA

17. A farmer lives near a straight river and wishes to pasture his sheep next to it, knowing they cannot cross the river. He has 800 meters of fence which he'd like to use to enclose the pasture. What is the maximum area, to the nearest square meter, he can enclose with his fence and the river?

(A) 8,000      (B) 97,142      (C) 101,859      (D) 11,578      (E) NOTA

18. How many complex roots does  $f(x) = x^3 - 2x^2 + 4x - 8$  have?

(A) 0      (B) 1      (C) 2      (D) 3      (E) NOTA

19. Given that  $4-2i$  is a root of  $x^3 - kx^2 + x - 5 = 0$ , which of the following are roots?

- I. 1
- II.  $\frac{1}{4}$
- III.  $4+2i$

(A) III      (B) II & III      (C) I & II      (D) I & III      (E) NOTA

20. Which of the following is a quadratic equation with roots  $2 \pm 3i$ ?

- I.  $x^2 - 4x + 13 = 0$
- II.  $x^2 + 4x - 7 = 0$
- III.  $2x^2 + 8x - 3 = 0$

(A) I      (B) II      (C) III      (D) II & III      (E) NOTA

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21. A cubic equation has  $3 - i$  as one of its roots, and no quadratic term. What is the constant term?
- (A) -60      (B) 60      (C) -54      (D) 54      (E) NOTA
22. Which of the following are subsets of the real numbers?
- I. the rational numbers  
II. the complex numbers  
III. the integers  
IV. the irrational numbers
- (A) I, II, III, & IV   (B) I, III, & IV   (C) I & IV      (D) I, II, & III      (E) NOTA
23. A cubic equation with integral coefficients has  $4 + 2\sqrt{3}$  as one of its roots, has a leading coefficient of 1, and a constant term of 8. What is the coefficient of the quadratic term?
- (A) -6      (B) 6      (C) -4      (D) 4      (E) NOTA
24. A real number is twice its reciprocal. What is the number?
- (A)  $\frac{1+\sqrt{5}}{2}$       (B)  $\frac{1+\sqrt{3}}{2}$       (C)  $\sqrt{3}$       (D)  $\sqrt{2}$       (E) NOTA
25. Find the value of the constant term in the expansion of  $\left(4x + \frac{3}{x^2}\right)^6$
- (A) 256      (B) 2304      (C) 3840      (D) 34560      (E) NOTA
26. If  $f(x) = 2x$  and  $g(x) = \sqrt{16 - x^2}$ , what is the domain of  $g(f(x))$ ?
- (A)  $|x| \leq 2$       (B)  $|x| \leq 4$       (C)  $x \leq 4$       (D)  $0 < x \leq 2$       (E) NOTA
27. For what values of  $a$  is  $a\sqrt{42} \neq \sqrt{42a^2}$ ?
- (A)  $a \leq 0$       (B)  $a < 0$       (C)  $a \in \mathfrak{R}$       (D)  $a < \sqrt{42}$       (E) NOTA
28. In the expansion of  $(x + 3)^n$  the coefficient of the third term is 27. What is  $n$ ?
- (A) 5      (B) 4      (C) 3      (D) 2      (E) NOTA

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29. What is the sum of the eight complex eighth roots of  $4 - 2i$ ?
- (A)  $4 - 2i$       (B) 0      (C) 1      (D)  $4 + 2i$       (E) NOTA
30. A rancher wishes to corral his cattle next to his barn, thus saving on fencing material. His barn is 80 meters long, and he buys 120 meters of fence at the general store. What is the maximum area, to the nearest square meter, he can enclose with a rectangular corral?
- (A) 1518      (B) 1642      (C) 1800      (D) 2157      (E) NOTA