Applications / Areas & volumes - 1999 Theta

1. C $e^3 = 6e^2$
$e = 6$

2. B $x^2\pi = 2a^2$

3. C $(4x)^3 - (3x)^3 = 296$
$64x^3 - 27x^3 = 296$
$37x^3 = 296$
$x^3 = 8$
$x = 2$
$x = 2$
$3x = \boxed{6}$

4. B $b^2 = \frac{1}{2}bh$
$2b^2 = bh$
$2b = h$

5. A $\frac{4}{3}\pi r^3 = 4\pi r^2$
$\frac{4}{3}\pi r = 4\pi$
$r = 4 \cdot \frac{3}{4}$
$\boxed{r = 3}$

6. C $V = \pi r^2 h$
$V = \pi(3)^2(6)$
$V = \pi(9)(6) = 54\pi$

7. B $V_{cone} = \frac{1}{3}\pi r^2 h$ $V_{cylinder} = \pi r^2 h$
$= \frac{\frac{1}{3}\pi r^2(h)}{\pi r^2(h)} =$ $\frac{1}{3} = \boxed{\frac{1}{3}}$

8. D $A = lw$ $2A = ba$
$A = ba$
$\frac{2A}{b} = a$

The area doubles. One length remains constant. The second length must double in order for the area to double.

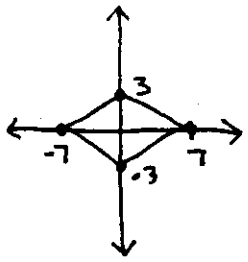
9. A $\frac{\text{Area Circle I}}{\text{Area Circle II}} = \frac{(r_1)^2}{(r_2)^2} = \frac{25}{16}$

as well

$\frac{5}{4} = \frac{r_1}{r_2}$ $\frac{d_1}{d_2} = \frac{5}{4}$
$5r_2 = 4r_1$ $4d_1 = 5d_2$
$\frac{5}{4}r_2 = r_1$ $d_1 = \frac{5}{4}d_2$

d_1 is 1.25 larger than d_2

10. B

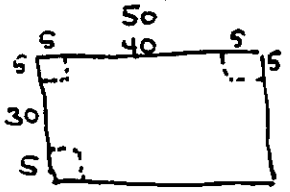


4 right triangles.
4. $\frac{1}{2} \cdot 3 \cdot 7$
$2 \cdot 3 \cdot 7$
42

11. C $\pi r^2 h = \pi r(2r + 2h)$
$rh = 2r + 2h$
$rh - 2h = 2r$
$h(r-2) = 2r$
$h = \frac{2r}{r-2}$

Since h must be positive, the smallest integral value of r is 3.

12. B



$V = 40 \cdot 30 \cdot 5$
$V = 6000$ cu in.

13. A $V = 24$ ft^3
$V = \frac{24}{3} = 8$ ft^3
$e = 2$

14. E $A = \frac{e^2\sqrt{3}}{4} = \frac{10^2\sqrt{3}}{4} = 25\sqrt{3}$

15. D $\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r^3$ $\frac{468\pi}{3}$
$\frac{4}{3}\pi(125) - \frac{4}{3}\pi \cdot 8$ $\boxed{156\pi}$
$\frac{500\pi}{3} - \frac{32\pi}{3}$

16. A $P = 800$ $s = \frac{P}{4} = \frac{800}{4} = 200$
$200^2 = \boxed{40{,}000 \text{ ft.}^2}$

17. C $\frac{s^3}{(s\sqrt{3})^3} = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{9}$

18. E $V = \frac{1}{3}\pi r^2 h$ Altitude of Δ is 4. radius is 3.
$V = \frac{1}{3}\pi(3)^2(4)$
$V = 12\pi$

19. D $\frac{4}{3}\pi r^3 - \pi r^2 h =$ $\frac{288\pi}{3}$
$\frac{500\pi}{3} - 72\pi =$ $\boxed{94\frac{2}{3}\pi}$
$\frac{500\pi}{3} - \frac{216\pi}{3} =$

20. A

$A = \frac{1}{2} d_1 d_2$

$15 = \frac{1}{2}(7)(d_2)$

$15 = \frac{7}{2} d_2$

$15 \cdot \frac{2}{7} = d_2$

$\frac{30}{7} = d_2$

21. B

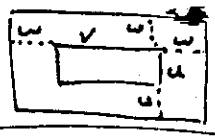
$x^2 + y^2 - 4x - 2y = 75$

$x^2 - 4x + 4 + y^2 - 2y + 1 = 75 + 4 + 1$

$(x-2)^2 + (y-1)^2 = 80 \quad r = \sqrt{80}$

$\pi r^2 = \pi(\sqrt{80})^2 = 80\pi$

22 D



$(u+2w)(v+2w) - uv$

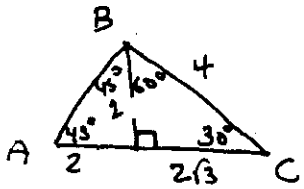
23. A.

Old volume: $\frac{1}{3}(4)(16) = \frac{64\pi}{3}$

New volume: $2 \cdot \frac{1}{3} \cdot 2 \cdot 16\pi = \frac{32\pi}{3} \cdot 2 = \frac{64\pi}{3}$

EXACTLY ENOUGH

24



$A = \frac{1}{2}(2)(2+2\sqrt{3})$

$A = (2+2\sqrt{3})$

$A = 2\sqrt{3} + 2$

25. B

$x - y = 4$

$x + y = 12$

$2x = 16$

$x = 8$

(8, 4) VERTEX of △.

x-INTERCEPT

$x - y = 4$

$-y = -x + 4$

$y = x - 4$

$0 = x - 4$

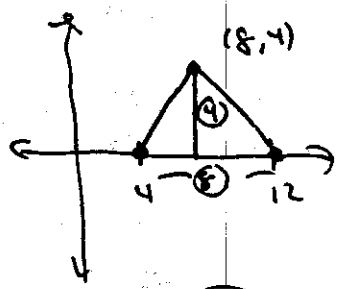
$4 = x$

$x + y = 12$

$y = -x + 12$

$-12 = -x$

$12 = x$



$A = \frac{1}{2}(8)(4) = (4)(4) = 16$

26) D

$36\pi = \frac{4}{3}\pi r^3$

$27 = r^3$

$r = 3$

$S = 4\pi r^2$

$= 4\pi(3)^2$

$= 36\pi$

27. E

$\frac{V = \frac{1}{3}\pi r^3}{V = \frac{2}{3}\pi r^3} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2}$

28. A

$A = \frac{6 \cdot e^2\sqrt{3}}{4}$

$\frac{3e^2\sqrt{3}}{2}$

$\frac{3 \cdot 8^2\sqrt{3}}{2} = 96\sqrt{3}$

29. A

$\frac{A}{18} = \left(\frac{1}{3}\right)^2$

$\frac{A}{18} = \frac{1}{9}$

$A = 2$

30. B

$\left(\frac{2}{3}V - \frac{2}{2}V = 6\right)6$

$4V - 3V = 36$

$V = 36$

T1.

$\left(\frac{5}{2}\right)^2 = \frac{A}{40}$

$\frac{25}{4} = \frac{A}{40}$

$A = 250$

Brahmagupta's.

T2

$S = \frac{2+7+6+9}{2} = 12$

$A = \sqrt{(12-2)(12-7)(12-6)(12-9)}$

$= \sqrt{10 \cdot 5 \cdot 6 \cdot 3}$

$= \sqrt{900}$

30

T3.

$V_{larger} - V_{smaller}$

$\frac{1}{3}Bh - \frac{1}{3}B'h'$

$\frac{1}{3}\pi 9^2 \cdot 12 - \frac{1}{3}\pi \cdot 6^2 \cdot 8$

228π