**Mu Alpha Theta National Convention 2004**
**CIRCLES**
**Answers**

| # | **Answer** | # | **Answer** |
|---|---|---|---|
| 1 | C | 18 | B |
| 2 | B | 19 | E |
| 3 | C | 20 | C |
| 4 | D | 21 | C |
| 5 | B | 22 | D |
| 6 | A | 23 | D |
| 7 | **D** | 24 | B |
| 8 | C | 25 | C |
| 9 | D | 26 | B |
| 10 | C | 27 | A |
| 11 | B | 28 | B |
| 12 | B | 29 | C |
| 13 | B | 30 | B |
| 14 | C | TB1 | $x^2 + y^2 + \frac{1}{3}x - \frac{7}{3}y - 8 = 0$ |
| 15 | B | TB2 | 9 |
| 16 | E | | |
| 17 | D | | |

## SOLUTIONS

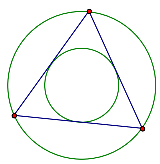
1. C $r^2 = \left(\frac{r\sqrt{k}}{2}\right)^2 + \left(r\sqrt{k}\right)^2 \rightarrow r^2 = \frac{kr^2}{4} + kr^2 \rightarrow 1 = \frac{k}{4} + k \rightarrow k = \frac{4}{5}$

2. B Slope of the radius $m = \frac{-3}{4}$ $\perp m = \frac{4}{3}$ $y = \frac{4}{3}x + 4 \rightarrow 4x - 3y = -12$

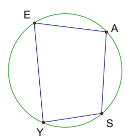
3. C Arc AC = 130° m∠ ABC = 65°

4. D 400 = 12(12 + 2x) $x = \frac{32}{3}$

5. B

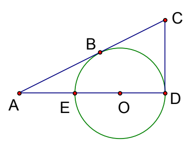


6. A



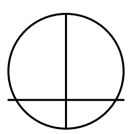

Since ∠ E = 75°, arc ASY = 150° arc EY = 360° - 275° = 85°

7. D



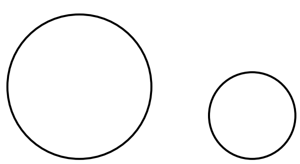

∠ AOB = 60° making arc BE 60° leaving arc BDE = 300°

8. C



$24^2 = 18x$ x = 32 radius = .5 (50) = 25

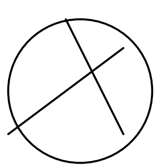
9. D



$\frac{240}{360} \cdot 60\pi + 2\left(24\sqrt{3}\right) + \frac{120}{360} \cdot 12\pi = 44\pi + 48\sqrt{3}$

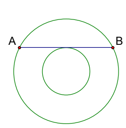
10. C $4x^2 + 4y^2 + 24x - 20y - 3 = 0$ $(x + 3)^2 + (y - 2.5)^2 = 16$ radius = 4 area = $16\pi$

11. B

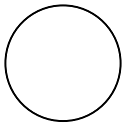


∠AXD = .5 (110 + 85) = 97.5 ∠AXC = 180 – 97.5 = 82.5

12. B



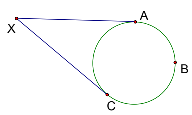

$10^2 + r^2 = R^2$ $R^2 - r^2 = 100$ area = $100\pi$

13. B  diameter = 5  circumference = $5\pi$

14. C $\frac{x}{6} = \frac{6}{13-x}$  $x^2 - 13x + 36 = 0 \rightarrow x = 4$ or 9  $4^2 + 6^2 =$ 52

$d_1 = 2\sqrt{13} \cdot 2 = 4\sqrt{13}$  $6^2 + 9^2 = 117$  $d_2 = 3\sqrt{13} \cdot 2 = 6\sqrt{13}$  $\frac{1}{2}\left(4\sqrt{13}\right)\left(6\sqrt{13}\right) = 156$

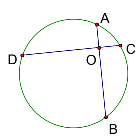
requested area is $156 - 36\pi$

15. B 

$5x = 360 \rightarrow x = 72 \rightarrow 4x = 288$  $\angle X = .5(288 - 72) = 108$

16. E needs picture  $\frac{4}{12} = \frac{x}{20-x}$  x = 5

17. D center of circle is (9, 8)  diameter = $\sqrt{45}$  $(x - 9)^2 + (y - 8)^2 = 45$

18. B 

let y be the distance between the center and chord AB then

$y^2 + 7^2 = r^2$  $(6 - y)^2 + 5^2 = r^2$  $y^2 + 7^2 = (6 - y)^2 + 5^2$  y = 1  $1 + 49 = r^2$

$r = 5\sqrt{2}$

19. E

20. C radius = $2\sqrt{10}$  circumference = $2\left(2\sqrt{10}\right)\pi = 4\sqrt{10}\pi$

21. C $\frac{36}{60} \cdot 30 = 18°$  $\frac{4}{5} \cdot 30 = 24°$  60° + 18° + 24° = 102°

22. D 360° - 195° = 165°  arc AD = 165°  $\angle$AXD = 82.5°  $\angle$AXC = 97.5°

23. D $x(14 - x) = 11\left(\frac{45}{11}\right)$  $x^2 - 14x + 45 = 0$  x = 5 or x = 9

24. B kite

25. C $2(4\pi r^2) = 8\pi$ $r^2 = 1$ $r = 1$ $h = 4$ $V = \pi r^2 h$ $V = 4\pi$

26. B $\pi r^2 = \left(\frac{p}{4}\right)^2$ $\pi r^2 = \frac{p^2}{16}$ $\frac{p^2}{r^2} = 16\pi$ $\frac{p}{r} = 4\sqrt{\pi}$

27. A The distance between the center and the tangent line is the radius.

$r = \frac{|8+9-7|}{\sqrt{16+9}} = 2$ $(x - 2)^2 + (y + 3)^2 = 4$

28. B $.5(64\pi) = 32\pi$ with the cone $8\pi r = 32\pi$ $r = 4$ $h = 4\sqrt{3}$

29. C $x^2 + 2x + 185 + 3x + 91 = 360$ $x^2 + 5x – 84 = 0$ $x = 7$ $112° - 35° = 77°$

$\angle QTR = 38.5°$

30. B Area = .5(12)(8 + 18) = 156

TB 1. $3x^2 + 3y^2 + x – 7y – 24 = 0$ Substituting the points into the general equation for a circle, you create a systems of equations: 2D – E + F = -5; -3D + F = -9; D + 4E + F = -17. Solve using matrices $D = \frac{1}{3}$, $E = \frac{-7}{3}$, and F = -8 Substituting and ridding the equation of fractions yields $x^2 + y^2 + \frac{1}{3}x – \frac{7}{3}y – 8 = 0$

TB 2. The crease bisects the radius. A radius to the end of the chord forms a 30-60-90 triangle with hypotenuse 18 and the distance from the center of the circle is the shortest leg of the triangle, or one-half the radius, 9.