1999 Mu Alpha Theta National Convention
Ciphering - Theta Division

1. Find the diameter of a sphere that has a volume numerically equal to its surface area.

Answer: 6
Solution: $\frac{4}{3}\pi r^3 = 4\pi r^2 \quad r = 3 \quad d = 6$

2. When the Tennessee Chapter of the Grizzly fan club quit accepting new members, it boasted a total of 500 members, 99% of which were male. One year later, no members had joined the club. However, some males, but no females, had withdrawn from the club and the club was then only 96% male. How many members were then in the club?

Answer: 125
Solution: Initially the female membership is 1%, or 5 people. Hence, when 5 females comprise 4% of the club, the total membership is 5/.04 = 125

3. Find the exact perimeter of Triangle ABC in terms of 'a' if A(- a, 0), B(a, 0) and C(0, $a\sqrt{3}$).

Answer: 6a
Solution: plot the points to see that it creates an equilateral triangle. P = a + a + 2a + 2a = 6a

4. Given $\frac{a}{b} = 3$, $\frac{a}{c} = 12$, and $\frac{a}{d} = 2$. Find the value of $\frac{ab}{cd}$

Answer: 8
Solution: $b = \frac{a}{3}$, $\frac{a}{12} = c$, $\frac{a}{2} = d$ $\quad \dfrac{a \cdot \frac{a}{3}}{\frac{a}{12} \cdot \frac{a}{2}} = a \cdot \frac{a}{3} \cdot \frac{12}{a} \cdot \frac{2}{a} = 8$

5. Given $7^3 \cdot 6^3 \cdot 15^2 = 21^x \cdot 14^y \cdot 10^z \cdot 3^w$, where x, y, z, and w ≠ 0. Find x + y + z + w.

Answer: 8
Solution: $7^3 \cdot 6^3 \cdot 15^2 = 21^x \cdot 14^y \cdot 10^z \cdot 3^w$;

$$7^3 \cdot 2^3 \cdot 3^2 \cdot 3^2 \cdot 5^2 = (3\cdot7)^x (2\cdot7)^y (2\cdot5)^z (3)^w$$
$$2^3 \cdot 3^5 \cdot 5^2 \cdot 7^3 = (3\cdot7)^2 \cdot (2\cdot7) \cdot (2\cdot5)^2 \cdot 3^3$$
$$= (21)^2 (14)(10)^2 (3)^3$$

x + y + z + w = 2 + 1 + 2 + 3 = 8

6. Solve for x if $|2 - |2 - |2 - |2 - x|||| = 0$

Answer: $x = 0, x = 4, x = -4, x = 8$
Solution: $2 - |2 - |2 - |2 - x||| = 0$; $|2 - |2 - |2 - x||| = 2$; $2 - |2 - |2 - x|| = 2$ or $2 - |2 - |2 - x|| = -2$

$-|2 - |2 - x|| = 0$ or $-|2 - |2 - x|| = -4$

$2 - |2 - x| = 0$ or $2 - |2 - x| = 4$ or $2 - |2 - x| = -4$

$|2 - x| = 2$ or $|2 - x| = -2$ or $|2 - x| = 6$

$2 - x = 2$ or $2 - x = -2$ or $2 - x = 6$ or $2 - x = -6$
$x = 0, x = 4, x = -4, x = 8$

7. Solve the following equation for x: $X^2 + 2AX - 3XY - 6AY = 0$

Answer: $x = -2a, x = 3y$
Solution: Factor: $x(x + 2a) - 3y(x + 2a) = 0$; $(x + 2a)(x - 3y) = 0$; $x = -2a$ or $x = 3y$

8. Find $f(664)$ if $f(x)$ is a linear function such that $f(-2) = -1$ and $f(-1) = -4$.

Answer: - 1999
Solution: Find the equation of the line between the points (- 2, - 1) and (- 1, - 4). $f(x) = -3x - 7$ then evaluate $f(664) = -1999$

9. The point P (t, 4) lies on the line m, $3x - ty = 10$. Find the equation of the line n, in $Ax + By = C$ form where $A > 0$, and A, B, and C are integers, which is perpendicular to the line m and passes through the point Q(- t, t).

Answer: $10x - 3y = 130$
Solution: Since the point P lies on the line m, then $3t - 4t = 10$; $t = -10$. This means line m is $3x + 10y = 10$ and the line perpendicular to line m through the point Q (10, -10) is $10x - 3y = C$; $10(10) - 3(-10) = 130 = C$
Line n then is $10x - 3y = 130$

10. Seventeen people enter a cribbage tournament. Each entry is to play the other entries exactly once. How many games will be played in ALL? (Cribbage is a traditional card game played by two people where the score is kept by moving pegs in holes on a board.)

Answer: 136 games
Solution: $_{17}C_2 = \dfrac{17!}{(17-2)!\,2!} = \dfrac{17!}{15!\,2!} = \dfrac{17 \cdot 16}{2} = 136$

11. Find the distance between the center of $X^2 + 4Y^2 + 6X - 8Y + 9 = 0$ and the center of $Y^2 - 4X^2 + 4Y + 24X - 41 = 0$ in simplest radical form.

Answer: $3\sqrt{5}$
Solution: The center of the ellipse is (- 3, 1) and the center of the hyperbola is (3, - 2). The distance between them is $3\sqrt{5}$

12. Evaluate $4^{2\log_8 7}$. Express your answer in simplest radical form.

Answer: $7(\sqrt[3]{7})$

Solution: $$4^{2\log_8 7} = (2^2)^{2\log_{2^3}(7)} = 2^{\frac{4}{3}\log_2(7)} = 2^{\log_2 7^{\frac{4}{3}}} = 7^{\frac{4}{3}} = 7(\sqrt[3]{7})$$

Practice Round

If y varies jointly as x and z and inversely as $\sqrt{w}$, and y = 12 when x = 2, z = 6, and w = 9, find y when x = 5, z = 7, and w = 25.

Answer: 21
Solution: $\frac{y\sqrt{w}}{xz} = k$, where k is the proportionality constant. $\frac{12 \cdot \sqrt{9}}{2 \cdot 6} = \frac{y\sqrt{25}}{5 \cdot 7}$; $y = 21$