

Mu Alpha Theta National Convention 2004
Algebra – Theta Division

For each question, NOTA means “none of the above are correct”

1. Solve for x: $5x + 7 = -3$
A. -2 B. -1 C. 1 D. 2 E. NOTA
2. The lines $2x - y = 3$ and $x - y = -2$ intersect at what point?
A. (1, 2) B. (2, 1) C. (3, 5) D. (5, 7) E. NOTA
3. Which of the following, when simplified, is not a polynomial?
A. $(x+1)(x+2)$ B. $|x| - (x^2 + |x|)$ C. $\sqrt[3]{x^3 + 1}$ D. $(1 - x^2)^2$ E. NOTA
4. The graph of $y = x^2 - 2x$ never passes through which quadrant?
A. I B. II C. III D. IV E. NOTA
5. Which of the following is equivalent to: $x^k y^k k^x k^y$?
A. $(x + y)^k k^{x+y}$ B. $(x + y)^k k^{xy}$ C. $(xy)^k k^{x+y}$ D. $(xy)^k k^{xy}$ E. NOTA
6. Simplify: $\frac{(x^3 + 1)(x^2 - 1)}{(x + 1)^2}$
A. $(x^2 - x - 1)(x + 1)$ B. $(x^2 + x - 1)(x + 1)$
C. $(x^2 - 2x + 1)(x - 1)$ D. $(x^2 + 2x + 1)(x - 1)$ E. NOTA
7. If the zeroes of $f(x)$ are 3 and 4, then what are the zeroes of $f(x^2)$?
A. 9 and 16 B. ± 9 and ± 16 C. $\sqrt{3}$ and 2 D. $\pm\sqrt{3}$ and ± 2 E. NOTA
8. The graph of $y^2 = 5x + 4$ has what shape?
A. hyperbola B. parabola C. ellipse D. cardioid E. NOTA

9. Find the sum: $\sqrt{3} + 1 + \frac{\sqrt{3}}{3} + \frac{1}{3} + \frac{\sqrt{3}}{9} + \dots$
- A. $\frac{5}{3}$ B. $\frac{3\sqrt{3}}{2}$ C. $\frac{\sqrt{3}+3}{2}$ D. $\frac{3\sqrt{3}-1}{2}$ E. NOTA

10. In $Ax^2 + Bx + C = 0$, if $A = B = C$, what are the two solutions of x ?
- A. $\frac{1 \pm i\sqrt{3}}{2}$ B. $\frac{-1 \pm i\sqrt{3}}{2}$ C. $\frac{1 \pm \sqrt{5}}{2}$ D. $\frac{-1 \pm \sqrt{5}}{2}$ E. NOTA

11. Factor $2x^2 - 2xy + 3x - 3y$.
- A. $(x - 2y)(x + 3)$ B. $(2x - 3y)(x + 1)$ C. $(2x + y)(x - 3)$ D. $(x - y)(2x + 3)$ E. NOTA

12. The letters in “GOOGLE” can be rearranged g different ways.
 The letters in “YAHOO” can be rearranged y different ways.
 Find $\frac{g}{y}$
- A. 1 B. $\frac{5}{3}$ C. 2 D. 3 E. NOTA

13. Solve for x : $(\log_{10} x)\log_{10} 5 + \log_{10} 4 = 2$.
- A. 25 B. 100 C. 500 D. 1000 E. NOTA

14. Let $f(x) = x^2 + 2x$. Which of the following is one of the inverses of $f(x)$?
- A. $-1 - \sqrt{x+1}$ B. $-1 + \sqrt{x-1}$ C. $1 - \sqrt{x+1}$ D. $1 + \sqrt{x-1}$ E. NOTA

15. How many ordered pairs of integers (x, y) satisfy $xy = 48$?
- A. 5 B. 10 C. 20 D. 40 E. NOTA

16.
$$\begin{array}{c}
 1 \\
 2 \ 1 \ 1 \\
 4 \ 3 \ 3 \ 1 \\
 7 \ 6 \ 4 \ 3 \ 1
 \end{array}$$
 Continue this pattern of labeling the entries in Pascal’s Triangle, up to down, left to right. What is the 50th entry?
- A. 1 B. 36 C. 126 D. 252 E. NOTA

17. What is the distance between the points $(-3, 0, -3)$ and $(5, 1, 1)$?
- A. 9 B. $4\sqrt{5}$ C. $3\sqrt[3]{3}$ D. $2\sqrt[3]{10}$ E. NOTA
18. Factor $2x^3 + 7x^2 - 5x - 4$.
- A. $(x-1)(2x+1)(x+4)$ B. $(x-1)(2x-1)(x-4)$
 C. $(x-2)^2(2x-1)$ D. $(x+2)^2(2x-1)$ E. NOTA
19. Let $i = \sqrt{-1}$. What is the real part in the expansion of $(1+2i)^4$?
- A. -24 B. -21 C. -8 D. 41 E. NOTA
20. Let $f(x) = x^3$. Evaluate $f(a+b) + f(a-b)$.
- A. $2a^3$ B. $a^3 - b^3$ C. $6a^2b + 2b^3$ D. $2a^3 + 6ab^2$ E. NOTA
21. Which of the following is a real number?
- A. π B. i C. ∞ D. $\frac{0}{0}$ E. NOTA
22. $2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 6^{\frac{1}{6}}$ is equivalent to which of the following?
- A. $2^{\frac{1}{12}} \times 3^{\frac{1}{18}}$ B. $2^{\frac{5}{6}} \times 3^{\frac{2}{3}}$ C. $2^{\frac{2}{3}} \times 3^{\frac{1}{2}}$ D. $2^3 \times 3^2$ E. NOTA
23. Let $f(x) = (x-2)^7(x+1)^3$. Evaluate $f(f(-1))$.
- A. 0 B. -128 C. 128 D. 1 E. NOTA
24. A car completes a trip with an average speed of 50 mph. If the car covered $\frac{2}{5}$ of the distance at 60 mph what was the average speed for the remaining $\frac{3}{5}$ of the distance?
- A. 35 mph B. 40 mph C. $43\frac{1}{3}$ mph D. 45 mph E. NOTA
25. The polynomial $2x^3 + kx^2 - 18x - 8$ is evenly divisible by $(x + \frac{1}{2})$. What is the value of k?
- A. -67 B. -3 C. 1 D. 5 E. NOTA

26. If $a, b, c,$ and d are consecutive terms of a positive arithmetic sequence that satisfy:
 $-a^2 + b^2 - c^2 + d^2 = 3(a + b + c + d)$, then find $d - c$.

- A. -2 B. -1 C. 2 D. 3 E. NOTA

27. What is the sum of the x-coordinates of the x-intercepts of the following equation:

$$\frac{(x-5)^2}{25} + \frac{(y+3)^2}{25} = 1$$

- A. -1 B. 1 C. 8 D. 10 E. NOTA

28. How many distinct permutations are there of the word ALABAMA that begin with the letter A?

- A. 105 B. 120 C. 210 D. 5040 E. NOTA

29. A nation's currency has two different coins that have values of m and n . What is the maximum value that cannot be obtained using any number of these two coins? (m and n are two relatively prime integers each greater than or equal to 2.)

- A. $mn - 1$ B. $mn - m - n$ C. $mn + m + n$ D. $mn + 1$ E. NOTA

30. How many lattice points are solutions to the system of inequalities $\begin{cases} y > x^2 - 4x \\ y < 4 - x^2 \end{cases}$?

- A. 9 B. 10 C. 11 D. 12 E. NOTA

TB1. $\log_8 1024 + \log_{27} 243 + \log_7 \left(\frac{1}{343} \right) = ?$

TB2. If $f(g(x)) = 3$ and $f(x) = (5x + 2)^{\frac{1}{3}}$, and $g(x) = \frac{(x^2 - 1)}{3}$, then what is the sum of all the possible values of x ?

TB3. Find the value of a given the following:

$$\begin{aligned} 2a + b + 2c &= 14 \\ 3a - 2b + 4c &= -3 \\ -4a + 3b - 6c &= 6 \end{aligned}$$