1999 MU ALPHA THETA CONVENTION
RELAYS ROUND 1
ALPHA/MU

Sample

1. Consider a cube of side length s. Let A be the shortest possible distance travelled from one corner of the cube to the opposite corner and let B be the distanced travelled from one corner of the cube to the opposite corner by moving on the outside surface of the cube. If x = B – A, then find x/s.

2. A circle has equation $x^2 + y^2 - 4x + 10y = 52$. Let A be the number of units in the circumference of the circle and B be the number of square units in the area of the circle. Find A + B.

3. Calculate the probability that the sum of the squares of two numbers is less than 4 if each number is between 0 and 4.

4. Find the position of a particle at t = e given that its acceleration at any positive t is given by $a(t) = \frac{1}{t}$, and the velocity and the position of the particle at t = 1 are both 2.

Question 1

1. Find the sum of all integral solutions to the inequality $|10x - 5| \leq 208$.

2. Given the terminal side of θ lies in the third quadrant and $\sin\theta = -\frac{40}{41}$, find sin 2θ - cos 2θ.

3. The letters in the word CONVENTION are each written on an index card. The cards are placed in a bag and three cards are selected at random, from the bag. Calculate the probability that at least one card has a vowel on it. without replacement

4. Find the x-intercept of the line tangent to the graph of $y = \tan^2 x$ at the point where $x = \pi/4$.

Question 2

1. Consider the graph of $x^2 + y^2 - 6x + 8y + 9 = 0$. Let (a, b) be the center of the graph and let c be the length of a tangent drawn from the point (9, -8) to the circle. Find a + b + c.

2. Solve the system: $\begin{matrix} 1024^{x^2} = 16^{-2y} \\ xy = -10 \end{matrix}$.

3. If $z = \sum_{k=1}^{1000} (2k-1)i^k$, $i = \sqrt{-1}$, calculate the magnitude of z.

4. R is the region bounded by the x-axis and the arch of y = cos x from x = -π/2 to x = π/2. Calculate the volume of the solid resulting from revolving R about the x-axis.

Question 3

1. Find all values of x for which $\begin{vmatrix} 3 & x \\ 2x & -2 \end{vmatrix} = -7(x+3)$.

2. Find the area of an ellipse with eccentricity 3/5 and focal width 64/5.

3. Find the sum of the coordinates of the point of discontinuity of $y = \dfrac{x^3+27}{x+3}$.

4. A rectangle symmetric with respect to the y-axis has two vertices lying on the x-axis and the other two on the graph of $y = e^{-|x|}$. Find the maximum possible area of the rectangle.

Question 4

1. Between the years 1990 and 2020, how many prime years occur?

2. Given that $3+\sqrt{2}$ and $2 + i$ both satisfy the equation $x^5 - 7x^4 + 6x^3 + 50x^2 - 139x + 105 = 0$, find the rational solution to the equation.

3. Find the sum of the positive integral factors of 1,400.

4. Find the slope of the curve $y^2 - 3xy - x^2 = 103$ at the fourth quadrant point where $x = 3$.

Question 5

1. Let $A = \begin{bmatrix} x & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$. If AB = [-4], find x.

2. Find the ~~positive difference~~ value(s) of [redacted] x such that $\dfrac{x!+(x+1)!}{(x-1)!-x!} = -\dfrac{48}{5}$.

3. Evaluate $3^{\frac{1}{0!}} \cdot 3^{\frac{1}{1!}} \cdot 3^{\frac{1}{2!}} \cdot 3^{\frac{1}{3!}} \cdot \Lambda$.

4. If $a_n = \sum_{k=0}^{n} k!$, find the last digit of $a_1 + a_2 + a_3 + \ldots + a_{1999}$.

1999 MU ALPHA THETA CONVENTION
RELAYS ROUND 2: GOLD DIVISION
ALPHA/MU

Sample

1. The sum of the cubes of two numbers is 728 and the sum of the numbers is 14. Find the positive difference of the numbers.

2. Find the area of the region enclosed by the graph of $4x^2 + 9y^2 - 24x + 72y + 144 = 0$.

3. Given that $\log 2 \approx 0.301$, calculate the number of digits in 5^{1999}.

4. Calculate the speed of a particle traveling along the curve $x(t) = -3\cos t$ and $y(t) = \sin t$ when $t = \pi/4$.

Question 1

1. The trace of a matrix is the sum of the entries on the main diagonal of the matrix. Find the trace of A if $A^{-1} = \begin{bmatrix} 7 & 2 \\ 6 & 2 \end{bmatrix}$.

2. When the Smiths bought their new house, the former owners left them 100 feet of fencing to enclose their backyard. Mr. Smith decided he would enclose a square area 25 feet by 25 feet since a square lot should maximize area. Mrs. Smith said his plot was not the maximum rectangular area that could be enclosed because only three sides of the yard would need to be fenced, and she proceeded to calculate the maximum possible rectangular area that could be enclosed with the fencing. How many **MORE** square feet are enclosed by her plan?

3. Find the volume of a cube inscribed in the sphere with equation $x^2 + y^2 + z^2 - 6x + 2y - 10z = 289$.

4. For the graph of $y = -\dfrac{x^2 - 9}{x^2 - 4}$, the intercepts are (a, 0), (b, 0), and (0, c). The asymptotes are x = d, x = e, y = f, and the graph is concave down on (g, h). Find abcdefgh.

Question 2

1. Find the constant term of the expansion of $\left(x^3 - \dfrac{1}{2x}\right)^{12}$.

2. Find the product of all x contained in the interval $(0, 2\pi]$ such that $\sin 3x = 2\sin^3 x$.

3. If $f(2x - 1) = x^2 - 3x + 7$, find the minimum value of f(x).

4. Evaluate $\displaystyle\int_{-1}^{1} \frac{x^4\,dx}{\sqrt{4 - x^2}}$.

Question 3

1. Find all complex solutions to $x^4 - 5x^2 - 36 = 0$.

2. A manufacturer produces units that sell for \$150 each. As an incentive for larger orders, the price per unit is reduced \$1 for each unit over 100 ordered. (If 110 units are ordered, each unit will cost \$140.) Find the order size that will maximize revenue.

3. Find the sum of the intercepts of the graph of $y = 65 - 16 \cdot 2^x - 4 \cdot 2^{-x}$.

4. On [-2, 2] the average value of $y = 4x^3 - 6x^2 + 8x + C$ is 20. Find C.

Question 4

1. In a system of equations in terms of x, y, and z, the value of x is 2 and can be calculated by evaluating the quotient $\dfrac{\begin{vmatrix} 0 & 4 & -2 \\ 4 & c & 3 \\ -1 & -6 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 4 & -2 \\ 2 & c & 3 \\ 3 & -6 & 5 \end{vmatrix}}$. Find c + y + z.

2. Simplify $\sqrt{2}\cdot\sqrt[4]{4}\cdot\sqrt[8]{8}\cdot\sqrt[16]{16}\cdot\Lambda$.

3. If $a = \sqrt{33 - 12\sqrt{6}}$ and $b = \sqrt{30 - 12\sqrt{6}}$, find $2a - b\sqrt{3}$.

4. Given x(0) = 8 and y(0) = -2 and given also that $3\dfrac{dy}{dt} + 2\dfrac{dx}{dt} = 10e^{2t}$ and $\dfrac{dy}{dt} - \dfrac{dx}{dt} = -5e^t$, find x(1) – y(1).

Question 5

1. Solve for x if $3251_x = 1532_{x+2}$.

2. The graph of y = A sin Bx + C has period 3π and contains the points (3π/4, -1) and (3π/2, 3). Find ABC.

3. Using a standard deck of shuffled cards, a hand of 4 cards is dealt. How many distinct hands have at least two cards with the same denomination (face value)?

4. If, for all nonzero x, $3f(2x) - 2f\left(\dfrac{1}{x}\right) = 12$, find f(1) · f(2).

1999 MU ALPHA THETA CONVENTION
RELAYS ROUND 2: BLUE DIVISION
ALPHA/MU

Sample

1. Find the sum of all integral x such that $12x^2 < 28x + 5$.

2. The zeroes of $y = x^3 + bx^2 + cx + d$ are ANS less than each of the zeroes of $y = x^3 - 4x^2 + x + 6$. Find $b + c + d$.

3. Find the value of x for which $\log_{ANS} x + \log_{ANS}(x - 24) = 2$.

4. In a group of ANS/3 students, there are twice as many upperclassmen as freshmen. If 3 students are selected at random, calculate the probability that all three are upperclassmen.

Question 1

1. Simplify completely: $\dfrac{x^3+8}{x^2-4} \div \dfrac{2x^2-4x+8}{4x-8}$.

2. The positive difference between the digits of a two digit number is ANS. When the digits are reversed, the new number is 50 less than twice the original number. Find the number.

3. Find the largest possible area of a rectangle of perimeter ANS.

4. Find the slope of the line tangent to the curve $y = 680\sqrt{x}$ at the point where x = ANS.

Question 2

1. Find the sum of the intercepts of the graph of $y = x^2 - 5x - 6$.

2. ANS is an x-intercept of the graph of $y = x^3 + x^2 + Ax + B$. Find $3A - B$ if $y(1) = 34$.

3. ANS people are invited to a party. The guests are asked to classify themselves as movie maniacs, cartoon clods, and/or music moguls. Of the guests, 4 classified themselves as all 3, 10 were both maniacs and moguls, 7 were maniacs that were also clods, 11 were fans of both tunes and toons, and 8 classified themselves as neither maniac nor clod nor mogul. How many guests classified themselves as exactly one type of fan?

4. ANS pounds are required to stretch a spring from its natural length of 10 inches to 18 inches. How many inch pounds of work are required to stretch the spring an additional 2 inches?

Question 3

1. Find the positive value of x for which $\frac{1}{x+1}$, x, and $2x^2 + 5x + 3$ form a geometric progression.

2. A committee of size ANS is to be chosen from a group of 5 men and 4 women. How many distinct committees will have exactly 2 women?

3. Given that $2 - i$ is a zero of $x^3 + 2x^2 - 19x + ANS$, find the only real root of the polynomial.

4. Find f(3) given that $f'(x) = x^2 - 6x$ and $f(0) = ANS$.

Question 4

1. Given $f(x) = \frac{3x^2 + 15x + 18}{x+2}$, find the sum of the coordinates of the point of discontinuity of the graph of f.

2. Evaluate $\frac{1}{27} \sum_{n=ANS}^{\infty} \left(\frac{3}{4}\right)^{n+2}$.

3. Find y – x if $32^{y-3x+\frac{4}{5}} = ANS$ and $9^{5x+1} = 243$.

4. Find the area of the region bounded by the x-axis, the graph of $y = x^2$, $x = ANS$, and $x = 2$.

Question 5

1. Find the sum of all θ in $[-\pi/2, \pi/2]$ such that $\cos 2\theta = -\sin \theta$.

2. Evaluate $\lim_{\beta \to ANS} \frac{\cos^3 \beta - \sin^3 \beta + \sin \beta - \cos \beta}{\cos \beta - \sin \beta}$.

3. Find the area of the region bounded by the graph of $x = |y|$ and the line $x = ANS$.

4. R is the first quadrant region bounded by the graph of $y = e^{-2ANS \cdot x}$ and the x-axis. Find the volume of the solid resulting from revolving R about the x-axis.