Mu Alpha Theta National Convention: Seattle, 1997
Open Gemini Test

1. Determine the equation of the ellipse with foci $(\pm 7,0)$ and passing through the point $(2,12)$.

(A) $\frac{x^2}{49}+\frac{y^2}{157}=1$ (B) $\frac{x^2}{98}+\frac{y^2}{196}=1$

(C) $\frac{x^2}{157}+\frac{y^2}{196}=1$ (D) $\frac{x^2}{196}+\frac{y^2}{157}=1$ (E) NOTA

2. Determine the equation of the parabola with focus (8,-3) and directrix $y=x$.

(A) $x^2+2xy+y^2-32x+12y+146=0$
(B) $2x^2+4xy+2y^2-32x+12y+73=0$
(C) $x^2+4xy+4y^2-32x+12y+73=0$
(D) $x^2+2xy+y^2-16x+6y+146=0$
(E) NOTA

3. In 3-space, the locus of points whose distance from the z-axis is equal to their height above the x-y plane is intersected by the plane $2x+y=10$. What is the shape of this intersection?

(A) a parabola (B) a hyperbola (C) an ellipse (D) not a conic section (E) NOTA

4. A farmer owns three non-overlapping square fields of areas 18, 20, and 26 acres. If each of the fields has a vertex in common with each of the other fields, what is the area of the triangle constrained between the three fields?

(A) $4\sqrt{5}$ (B) 9 (C) $\sqrt{95}$ (D) 10 (E) NOTA

5. A sheet of paper 40 cm by 70 cm is folded once so that opposing corners are coincident. What is the length of the crease in the paper?

(A) $5\sqrt{65}$ (B) $\frac{40\sqrt{65}}{7}$ (C) $\frac{13\sqrt{65}}{2}$ (D) $\frac{21\sqrt{65}}{4}$ (E) NOTA

6. Consider the locus of all points twice as far from the point A(3,7) as they are from the point B(10,3). What is the maximum distance a point on that locus can be from the line through A and B?

(A) $\frac{\sqrt{65}}{4}$ (B) $\frac{2\sqrt{65}}{3}$ (C) $\frac{3\sqrt{65}}{4}$ (D) $\frac{4\sqrt{65}}{3}$ (E) NOTA

7. Pete rows upstream for 3 hours. He then rows downstream to his starting point in 20 minutes less than it would have taken in still water. What is the largest number of minutes his round trip could have taken in still water?

   (A) $120+50\sqrt{15}$ (B) $180+45\sqrt{10}$ (C) $200+40\sqrt{7}$ (D) 320 (E) NOTA

8. Two squares differ by 11025. What is the smallest possible value of the smaller of the two squares?

   (A) 1296 (B) 1681 (C) 1849 (D) 2209 (E) NOTA

9. In a car race, there are only two competitors, and one of them is very confused. One of the cars starts driving clockwise around the 5 km long circular track at 350 kmph, while the other car drives counterclockwise at 250 kmph. How many times will the competitors have passed one another (discounting their initial and final collocation) by the time they meet at the starting line for the third time (not counting their initial collocation)?

   (A) 35 (B) 36 (C) 41 (D) 42 (E) NOTA

10. What is the volume of a pyramid whose base is an equilateral triangle with sides of length 6, and all of whose other edges are of length 10?

    (A) $12\sqrt{33}$ (B) 72 (C) $15\sqrt{35}$ (D) $25\sqrt{7}$ (E) NOTA

11. When one liter of water is added to an acidic solution, the resulting solution is 20% acid by volume. If 1 liter of pure acid is then added, the resulting solution becomes 28% acid. What is the volume of pure acid present in the original solution?

    (A) $\frac{9}{5}$ (B) 2 (C) $\frac{11}{5}$ (D) $\frac{23}{10}$ (E) NOTA

12. In the expansion of $(x+y)^{12}$, the second and third terms are equal when evaluated for x = p and y = q, where p and q are positive and $p+q=1$. What is the value of q?

    (A) $\frac{2}{13}$ (B) $\frac{1}{5}$ (C) $\frac{2}{9}$ (D) $\frac{1}{4}$ (E) NOTA

13. Each side of a regular octahedron is to be painted one of two colors. How many distinguishable patterns could be produced?

    (A) 19 (B) 20 (C) 21 (D) 22 (E) NOTA

14. How many natural numbers less than 1000 contain both a 2 and a 3 among their digits?

(A) 56 (B) 58 (C) 60 (D) 62 (E) NOTA

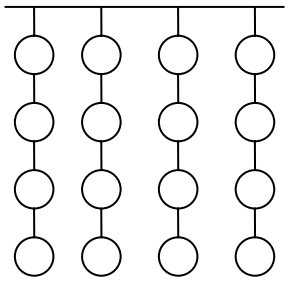
15. The measures of the interior angles of a convex polygon are in arithmetic sequence with a common difference of 5º. What is the maximum number of sides of the polygon?

(A) 56 (B) 58 (C) 60 (D) 62 (E) NOTA

16. A wooden cube of edge length 8 has a square hole drilled perpendicularly through each pair of opposing faces. If the edge length of these holes is 3, what is the maximum surface area of the sculpture, including interior surfaces?

(A) 606 (B) 610 (C) 614 (D) 618 (E) NOTA

17. Consider 4 strings, each supporting four glass balls, as shown in the diagram. In how many ways can all of these balls be broken one at a time, if only the balls which are the lowest on their string are eligible for breaking at any given moment?



(A) 59,427,000 (B) 60,606,000 (C) 61,276,000 (D) 63,063,000 (E) NOTA

18. Let A be the largest possible set of consecutive natural numbers which sum to 288. Let B be the product of all the elements of A. How many factors does B have?

(A) 3240 (B) 3600 (C) 3842 (D) 4320 (E) NOTA

19. A circle with diameter 30 cm circumscribes non-overlapping circles of diameters 10 and 20 cm. What is the diameter of the largest circle which can be inscribed within the 30 cm circle yet external to the other two?

(A) 9 (B) $\frac{60}{7}$ (C) $\frac{49}{6}$ (D) $\frac{42}{5}$ (E) NOTA

20. If the roots of $x^2 + px + q = 0$ are the cubes of the roots of $2x^2 - 7x - 3 = 0$, what is the sum of p and q?

(A) -62 (B) $-\frac{469}{8}$ (C) $-\frac{423}{8}$ (D) $-\frac{193}{4}$ (E) NOTA

21. Two balls, each 4 cm in radius, are situated in the corner of a rectangular room such that each is tangent to a wall, the floor, and the other ball. A third ball of radius 4 cm is then placed tangent to each of the two and to two of the walls. What is the maximum height of a point on the surface of a ball?

(A) $8 + 4\sqrt{2}$ (B) $4 + 8\sqrt{2}$ (C) 12 (D) $4 + 12\sqrt{2}$ (E) NOTA

22. As you wake up at the end of your math class, the teacher is saying that you must determine all roots of the equation on the chalkboard as your assignment for the next day, but the equation is in the process of being erased! All you are able to discern, as the equation is erased, is that it was a 20th degree polynomial with no leading coefficient, a constant term of 1, and a coefficient of -20 for the x^{19} term. What is the largest possible root of the equation?

(A) 0 (B) 1 (C) 32 (D) 1024 (E) NOTA

23. Determine $y'\left(\frac{\pi}{3}\right)$ if $y = x^{\cos x}$.

(A) $\frac{\sqrt{3\pi}}{3}\left[\frac{3}{\pi} + \frac{1}{2}\ln\left(\frac{3}{\pi}\right)\right]$ (B) $\frac{\sqrt{3\pi}}{3}\left[\frac{\sqrt{3}}{\pi} + \frac{1}{2}\ln\left(\frac{3}{\pi}\right)\right]$

(C) $\frac{\sqrt{3\pi}}{3}\left[\frac{1}{2\pi} + \frac{3}{4}\ln\left(\frac{3}{\pi}\right)\right]$ (D) $\frac{\sqrt{3\pi}}{3}\left[\frac{3}{2\pi} + \frac{\sqrt{3}}{2}\ln\left(\frac{3}{\pi}\right)\right]$ (E) NOTA

24. For what value of a will $x^2 - x + a$ evenly divide $x^{13} + x + 90$?

(A) -1 (B) 1 (C) -2 (D) 2 (E) NOTA

25. If N consecutive numbers, the lowest of which is 281, sum to N^3, what is the largest of the consecutive numbers?

(A) 297 (B) 311 (C) 318 (D) 324 (E) NOTA

26. What is the area of the region in the first quadrant contained by the polar equation $r = \sin 6\theta$?

(A) $\frac{\pi}{48}$ (B) $\frac{\pi}{24}$ (C) $\frac{\pi}{16}$ (D) $\frac{\pi}{12}$ (E) NOTA

27. Two 65 m long ladders lean against a building. The foot of one is 25 m from the building, while the foot of the other is 52 m from the building. How many meters above the ground do the two ladders cross?

(A) $\frac{324}{11}$ (B) 27 (C) $\frac{303}{10}$ (D) $\frac{117}{4}$ (E) NOTA

28. A squishy rectangle always remains rectangular and maintains a constant area. If the height is 5 cm and the width is 4 cm at the moment that the height is increasing by 1 cm/second, at what rate is the width decreasing, in cm/second?

(A) $\frac{4}{5}$ (B) $\frac{2}{3}$ (C) $\frac{3}{5}$ (D) $\frac{16}{25}$ (E) NOTA

29. What is the volume of the region enclosed by the rotation of $y = -(x-4)^4 + 16$ about the x-axis?

(A) $\frac{3642\pi}{5}$ (B) $\frac{10423\pi}{15}$ (C) $\frac{32768\pi}{45}$ (D) $\frac{98206\pi}{135}$ (E) NOTA

30. What is the volume of the solid by the rotation of the graph of $x^2 + y^2 - 10x + 24y + 120 = 0$ about the x-axis?

(A) $\frac{19289\pi^2}{18}$ (B) $1176\pi^2$ (C) $\frac{3418\pi}{3}$ (D) $\frac{6941\pi}{6}$ (E) NOTA