Analytic Geometry — National MAO — 2000

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d 1. d

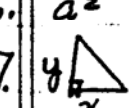
d 2. $m = \frac{a^3-b^3}{b-a} = -\frac{(a-b)(a^2+ab+b^2)}{b-a}$ $m_\perp = \frac{1}{a^2+ab+b^2}$

b 3. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $\pi ab = 14\pi$ $a=2$ $\frac{x^2}{4}+\frac{y^2}{49}=1$ length of minor axis $b=7$

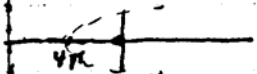
a 4. $(-6,y)$ $\sqrt{(7)^2+(3-y)^2}=\sqrt{74}$ $49+9-6y+y^2=74$ $y^2-6y-16=0$ sum of roots is 6.

b 5. $\frac{-3}{k}=\frac{5}{1}$ $5k=-3$ $k=-3/5$ $(-3/3,0)$ $(-h/5,0)$ $-3/3=\frac{-h}{5}$ $-3h=-10$ $h=\frac{10}{3}$ $-\frac{3}{3},\frac{10}{3}=-2$

a 6. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $c(0,0)$ $m=\pm\frac{b}{a}=\pm\frac{5}{12}=\pm\frac{10}{24}$ $c^2=a^2+b^2$ $c=26$ $a=24$

c 7.  $x+y=5$ $A=\frac{1}{2}xy$ $A(x)=\frac{1}{2}x(5-x)$ $A(x)=\frac{5}{2}x-\frac{1}{2}x^2$ Vertex $\frac{-b}{2a}$ $\frac{-5/2}{2(-1/2)}$ $x=5/2$ $A=\frac{1}{2}(5/2)(5/2)=25/8$

c 8.

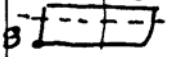
c 9.  directrix at $x=-8$ $r\cos\theta=-8$

a 10. $x^2+y^2=2500$ $x=110-2y$ $(110-2y)^2+y^2=2500$ $12100-440y+5y^2=2500$ $5y^2-440y+9600=0$ $y^2-88y+1920=0$ $(y-48)(y-40)=0$ $(30,40)$ $(14,48)$

$d=\sqrt{16^2+48^2}$ $d=\sqrt{320}$ $\frac{\sqrt{320}}{240}=.074535$ hr. $\times 60$ 4.47 minutes

c 11. $2ky=-9$ $y=-9/2k$ $\frac{-9}{2k}=-6$ $-12k=-9$ $k=3/4$

b 12. $B^2-4AC=0$ $B^2=4(-5)(-2)$ $B^2=40$ $B=2\sqrt{10}$

a 13.  $D(9,2)$ $B(-12,-5)$ $l+w=28$ $l=3w$ $4w=28$ $w=7$ $l=21$

b 14. If a $\triangle$ has 3 congruent medians it is equilateral.

d 15. hyperbola

a 16.  $h=\sqrt{16-x^2}$ $b_1=2x$ $b_2=8$ $\frac{1}{2}\sqrt{16-x^2}(2x+8)$ $(x+4)\sqrt{16-x^2}$

d 17. $x-1=e^t$ $t=\ln(x-1)$ $y=\sin(\ln(x-1))$

b 18. $r\left[\sin\theta\cos\frac{\pi}{6}+\cos\theta\sin\frac{\pi}{6}\right]=2$ $r\frac{\sqrt{3}}{2}\sin\theta+r\frac{1}{2}\cos\theta=2$ $x+\sqrt{3}y=4$

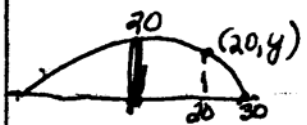
a 19. $3x-4y=17$ $3x+4y=1$ $6x=18$ $x=3$ $y=-2$ $\frac{(x-3)^2}{a^2}-\frac{(y+2)^2}{b^2}=1$ $(7,-2)$ $a^2=16$ $\frac{16}{a^2}=1$ $\pm\frac{b}{a}=\pm\frac{3}{4}$ $b=3/4(a)$ $b=3$ $\frac{(x-3)^2}{16}-\frac{(y+2)^2}{9}=1$ $\frac{(x-3)^2}{16}-\frac{4}{9}=1$ $\frac{(x-3)^2}{16}=\frac{13}{9}$ $x=-1.8$ $x=7.8$

c 20. Because of the symmetry of the area the line must pass through $(2,4)$
$\therefore 28+12=k$ $k=40$

c 21. $-\frac{a}{b}=\frac{-7}{4}$ $4a=7b$ $a=7b/4$ $\frac{7b}{4}+\frac{4b}{4}=22$ $11b=88$ $b=8$ $14i+8j=\vec{v}$ $\|\vec{v}\|=\sqrt{260}=2\sqrt{65}$

a 22. $\frac{|2(4)+3(-6)-6(1)-21|}{\sqrt{4+9+36}}=\frac{18}{7}$

c 23. $C=\left(\frac{5}{2},\frac{5\sqrt{3}}{2}\right)$ $r=3$ $\left(x-\frac{5}{2}\right)^2+\left(y-\frac{5\sqrt{3}}{2}\right)^2=9$ $x^2-5x+\frac{25}{4}+y^2-5\sqrt{3}y+\frac{25}{4}=9$ $x^2+y^2-5x-5\sqrt{3}y+16=0$
$r^2-10r\cos(\theta-60^\circ)+16=r^2-10[\cos\theta\cos 60+\sin\theta\sin 60^\circ]+16$

b 24. 

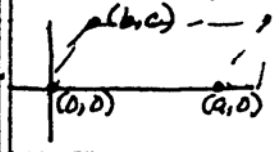
$\frac{x^2}{20^2}+\frac{y^2}{20^2}=1 \quad \frac{400}{900}+\frac{y^2}{400}=1 \quad \frac{y^2}{400}=\frac{500}{900}$

$900y^2=200000 \quad y^2=\frac{2000}{9} \quad y=\frac{20\sqrt{5}}{3}$

c 25. $y=\frac{x^3-8x^2+20x-14}{2(x-2)^2}$ i) F $x=2$ ii) T

$$\begin{array}{r|l} & \frac{1}{2}x-2+\frac{2}{2x^2-8x+8} \\ 2x^2-8x+8 & x^3-8x^2+20x-14 \end{array}$$

iii) T $(0,-7/4)$ iv) T v) F

d 26. 

$\frac{c}{b-a}\cdot\frac{c}{a+b}=-1$

d 27. $z_1z_2=r_1r_2\,\mathrm{cis}(\alpha+\beta)$

c 28. $y(y-3x)=0 \quad y=0 \quad y=3x$

a) 29. $\tan(31.4^\circ)=m \quad m=.6104 \quad m_\perp=-1.64$

c 30. The change from A to B and D to C is (a,b,c), the change from B to C and A to D is $(\frac{1}{a},\frac{1}{b},-\frac{2}{c})$ and $(a^2+b^2+c^2)+(\frac{1}{a^2}+\frac{1}{b^2}+\frac{4}{c^2})=(a^2+\frac{1}{a^2}+2)+(b^2+\frac{1}{b^2}+2)+(c^2+\frac{4}{c^2}-4)$ ∴ by Pythagorean Theorem the angles are right angles.