2

(1) the distance between a point on a line and a second line is given by the formula $d = \dfrac{|ax+by+c|}{\sqrt{a^2+b^2}}$ where $ax+by+c=0$.

if $x=1$ then $x+2y+4=0$
$1+2y+4=0$
$2y=-5$
$y=-\frac{5}{2}$

$(1,-\frac{5}{2})$ and $2x+4y-5=0$

$$d = \frac{|2(1)+4(-\frac{5}{2})+-5|}{\sqrt{2^2+4^2}} = \frac{|2+-10+-5|}{\sqrt{20}} = \frac{|-13|}{2\sqrt{5}} = \frac{13}{2\sqrt{5}} = \frac{13\sqrt{5}}{10} \quad \boxed{B}$$

IF $v = ai+bj$ and $w = ci+dj$

(2) then $|v|\cos\theta = \dfrac{ac+bd}{|w|}$ where $|w| = \sqrt{c^2+d^2}$

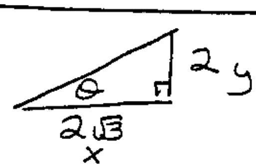
So $|v|\cos\theta = \dfrac{12(7)+(-5)(24)}{\sqrt{7^2+24^2}} = \dfrac{84+-120}{\sqrt{625}} = -\dfrac{36}{25} \quad \boxed{A}$

(3) $||v-w|| = ||4i-4j|| = \sqrt{16+16} = \sqrt{32}$

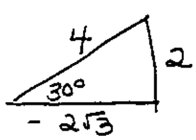
$||v+w|| = ||(3-1)i+(2+6)j|| = ||2i+8j|| = \sqrt{2^2+8^2} = \sqrt{4+64} = \sqrt{68}$

So $||v-w||^2 + ||v+w||^2 = \sqrt{32}^2 + \sqrt{68}^2 = 32+68 = 100 \quad \boxed{C}$

(4) $(2\sqrt{3}, 2) = (x, y)$




$\Rightarrow 30°-60°-90° \triangle$




$r=4 \Rightarrow \tan\theta = \frac{y}{x} = \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Since $(2\sqrt{3}, 2)$ is in Quadrant I $\Rightarrow \tan\theta = \frac{\sqrt{3}}{3} \Rightarrow \theta = 30°$ or $\frac{\pi}{6}$

So the polar coordinates are $(r,\theta) = (4, \frac{\pi}{6}) \quad \boxed{A}$

(5) $\sqrt{2-2x-x^2}$
$= \sqrt{-(x^2+2x\quad)+2}$
$= \sqrt{-(x^2+2x+1)+2+1}$
$= \sqrt{-(x+1)^2+3}$
$\Rightarrow -(x+1)^2+3 \geq 0$
$-(x+1)^2 \geq -3$

$(x+1)^2 \leq 3$
$x+1 = \pm\sqrt{3}$
$x = -1 \pm \sqrt{3}$ are the critical pts


$\{-1-\sqrt{3} \leq x \leq -1+\sqrt{3}\} \quad \boxed{E}$

(6) By the midline theorem the length of the midlines are equal to 1/2 the length of the side || to the midline so finding the perimeter of $\frac{1}{2}\triangle ABC$ = the perimeter of the triangle formed by the joining of the midpoints of the sides

$AB = \sqrt{(-2+1)^2+(-1-4)^2} = \sqrt{(-1)^2+(-5)^2} = \sqrt{1+25} = \sqrt{26}$

$BC = \sqrt{(-1-3)^2+(4-2)^2} = \sqrt{(-4)^2+(2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

$AC = \sqrt{(-2-3)^2+(-1-2)^2} = \sqrt{(-5)^2+(-3)^2} = \sqrt{25+9} = \sqrt{34}$

perimeter $= \frac{1}{2}(\sqrt{26}+2\sqrt{5}+\sqrt{34}) \approx \frac{1}{2}(5+2(2\ )+6) \approx \frac{1}{2}(15) = 7.5$

$[7.5] = 7$ $\boxed{B}$

(7) Using Euler's Formula:

# of Vertices + # of Faces − 2 = # of Edges

12 + 20 − 2 = # of Edges

30 = # of Edges $\boxed{B}$

(8) To find the intercepts of the equation set $x=0$ & solve for $y$ then $y=0$ & solve for $x$.

$y^4-4y^2 = 0^2+4(0)$

$y^2(y^2-4) = 0$

$y^2(y-2)(y+2) = 0$

$\Rightarrow y = 0, 2, \text{or } -2$

$(0,0)\ (0,2)\ (0,-2)$

$0^4-4(0)^2 = x^2+4x$

$0 = x(x+4)$

$x=0$ or $x=-4$

$(0,0)$ or $(-4,0)$

So the y intercepts are 0, 2 & −2 } the sum of the intercepts is
the x intercepts are 0 & −4 } $0+2+{}^-2+0+{}^-4 = {}^-4$ $\boxed{A}$

(9) Let $x = r\cos\theta$ and $y = r\sin\theta$ then $x^2+y^2+2x+6y=0$

$\Rightarrow (r\cos\theta)^2+(r\sin\theta)^2+2(r\cos\theta)+6(r\sin\theta)=0$

$r^2\cos^2\theta+r^2\sin^2\theta+2r\cos\theta+6r\sin\theta=0$

$r^2(\cos^2\theta+\sin^2\theta)+2r\cos\theta+6r\sin\theta=0$

$r(r+2\cos\theta+6\sin\theta=0$

$r+2\cos\theta+6\sin\theta=0$ or $r=0$

if $r=0$ then $\theta = \frac{\pi}{2}$

so $r+2\cos\theta+6\sin\theta=0$ $\boxed{C}$

(10) $9x^2+25y^2+18x-100y-116=0$

$9x^2+18x \quad +25y^2-100y = 116$

$9(x^2+2x \quad )+25(y^2-4y \quad ) = 116$

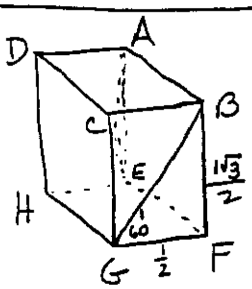
$9(x^2+2x+1)+25(y^2-4y+4) = 116+9(1)+25(4)$

$$\frac{9(x+1)^2}{9\cdot 25}+\frac{25(y-2)^2}{9\cdot 25}=\frac{225}{9\cdot 25}$$

$$\frac{(x+1)^2}{25}+\frac{(y-2)^2}{9}=1$$

$a^2=25 \Rightarrow a=5$ $\quad b^2=9 \Rightarrow b=3$

So the length of the major axis is $2(a)=2(5)=10$ and the length of the minor axis is $2(b)=2(3)=6$ $\therefore$ the sum of the squares of the lengths of the minor & major axes is $10^2+6^2=100+36=136$ $\boxed{B}$

(11) 


Let $BG=1$. So then $GF=BC=AD=EH=\frac{1}{2}$

and $BF=CG=DH=DC=HG=AB=EF=AE=\frac{\sqrt{3}}{2}$

$\Rightarrow \triangle DCB \cong \triangle GCB \Rightarrow DB=1$.

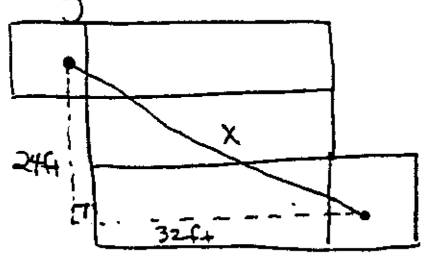
$$DG=\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\sqrt{\frac{3}{4}+\frac{3}{4}}=\sqrt{\frac{8}{4}}=\frac{\sqrt{6}}{2}$$

By the law of cosines $BD^2=BG^2+DG^2-2(BG)(DG)\cos\angle DGB$

$$\Rightarrow \cos\angle BGD=\frac{BD^2-BG^2-DG^2}{-2BG\,DG}=\frac{1^2-1^2-\frac{\sqrt{6}}{2}}{-2(1)(1)}=\frac{1-1-\frac{\sqrt{6}}{2}}{-2}=\frac{-\frac{\sqrt{6}}{2}}{-2}=\boxed{\frac{\sqrt{6}}{4}}$$

$\boxed{D}$

(12) Unfolding the room we have




$\sqrt{24^2+32^2}=\sqrt{576+1024}=\sqrt{1600}=40ft$

$\boxed{C}$

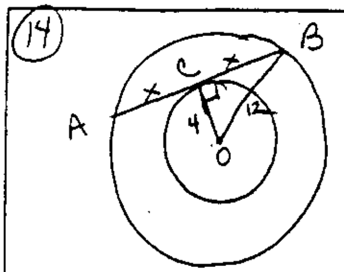
(13) $Ax^2+By^2+Cx+Dy+E=0$ can be written as the difference of 2 squares equalling a particular value

$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ or $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$

because $A \neq B \neq 0$ & if $A>0$ then $B<0$ or if $B>0$ then $A<0$

so the graph is a Hyperbola with center $(h,k)$ $\boxed{D}$

(14)

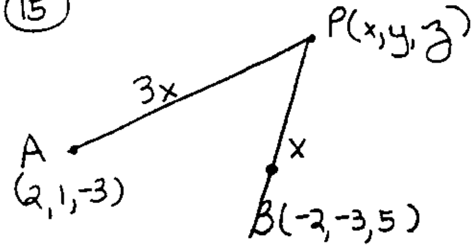



$\overline{AB}$ is tangent to the smaller circle and a chord of the larger circle, $\overline{AB} \perp \overline{OC}$. and $\overline{OC}$ bisects $\overline{AB}$

$x = \sqrt{12^2 - 4^2} = \sqrt{144-16} = \sqrt{128} = 8\sqrt{2}$

$AB = 2x = 2(8\sqrt{2}) = 16\sqrt{2}$ $\boxed{E}$

(15)




$AP = \sqrt{(2-x)^2 + (1-y)^2 + (-3-z)^2} = 3x$

$BP = \sqrt{(-2-x)^2 + (-3-y)^2 + (5-z)^2} = x$

$(AP) = 3(BP)$

$\sqrt{(2-x)^2 + (1-y)^2 + (-3-z)^2} = 3\sqrt{(-2-x)^2 + (-3-y)^2 + (5-z)^2}$

$(2-x)^2 + (1-y)^2 + (-3-z)^2 = 9(-2-x)^2 + 9(-3-y)^2 + 9(5-z)^2$

$4 - 4x + x^2 + 1 - 2y + y^2 + 9 + 6z + z^2 = 9[4 + 4x + x^2 + 9 + 6y + y^2 + 9 + 25 - 10z + z^2]$

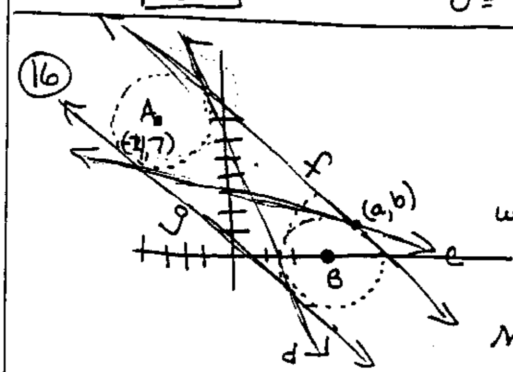
$x^2 - 4x + y^2 - 2y + z^2 + 6z + 14 = 9[x^2 + 4x + y^2 + 6y + z^2 - 10z + 38]$

$= 9x^2 + 36x + 9y^2 + 54y + 9z^2 - 90z + 342$

$0 = 8x^2 + 40x + 8y^2 + 56y + 8z^2 - 96z + 328$

$\boxed{B}$ $0 = x^2 + 5x + y^2 + 7y + z^2 - 12z + 41$

(16)




$\bigcirc_A$ $(x+4)^2 + (y-7)^2 = 2$ $\bigcirc_B$: $(x-3)^2 + y^2 = 2$

$\Rightarrow C(-4, 7)$ $r = \sqrt{2}$ $\Rightarrow C(3, 0)$ $r = \sqrt{2}$

with the ~~external~~ common tangents of c, d, f, & g

f has the largest y-intercept.

$m_f = m_{AB} = \dfrac{7-0}{-4-3} = \dfrac{7}{-7} = -1$

To find a point on this line it must also be on the 2nd circle

So call the point $(a,b)$ with $(x-3)^2 + y^2 = 2$ and $m_{B(a,b)} = \dfrac{b-0}{a-3} = 1$

$(a-3)^2 + b^2 = 2$

because $f \perp B(a,b)$ so solving ~~for~~ $b = a-3$ ; $b^2 + b^2 = 2$

$2b^2 = 2$

$b^2 = 1$

$\Rightarrow b = 1$

$1 = a - 3 \Rightarrow 4 = a$

So a point on the line is $(4, 1)$

$y = -1x + b$

$1 = -1(4) + b$

$5 = b$

So the equation of the line is $y = -1x + 5 \Rightarrow x + y = 5$ $\boxed{C}$

(17) The equation of the circle is of the form $x^2+y^2+Dx+Ey+F=0$

$(-3,1)$: $Dx+Ey+F = Dx^2+Ey^2$

$-3D+1(E)+F = (-3)^2+1^2$

$-3D+E+F = 10$

$(-2,2)$: $Dx+Ey+F = Dx^2+Ey^2$

$-2D+2E+F = (-2)^2+2^2$

$-2D+2E+F = 8$

$(6,-2)$: $Dx+Ey+F = Dx^2+Ey^2$

$6D+{}^-2E+F = (6)^2+(-2)^2$

$6D+{}^-2E+F = 40$

Solving the system:

$-3D+E+F = 10$

$-2D+2E+F = 8$

$6D+{}^-2E+F = 40$

$$\begin{array}{r} -3D+E+F=10 \\ -(-2D+2E+F=8) \\ \hline D+{}^-E=2 \end{array} \qquad \begin{array}{r} -2D+2E+F=8 \\ -(6D+{}^-2E+F=40) \\ \hline -8D+4E=-32 \\ -2D+E=-8 \end{array}$$

$$\begin{array}{r} D+{}^-E=2 \\ -2D+E=-8 \\ \hline -D=-6 \\ D=6 \end{array} \qquad \begin{array}{r} D+{}^-E=2 \\ 6+{}^-E=2 \\ {}^-E=-4 \\ E=4 \end{array}$$

$-3(6)+4+F=10$

$F=24$

$\therefore x^2+y^2+6x+4y+24=0$ $\boxed{C}$

(18) $x^2+y^2-16x+12y=0$

$x^2-16x+y^2+12y=0$

$x^2+{}^-16x+64+y^2+12y+36=64+36$

$(x-8)^2+(y+6)^2=100$ $C(8,6)$ $r=10 \Rightarrow r=5$ to satisfy midpoint

$(x-8)^2+(y+6)^2=25$

$(0,\sqrt{60})$: $(-8)^2+(\sqrt{60}+6)^2$

$64+36+12\sqrt{60}+60 \neq 25$

$(4,-8)$: $(4-8)^2+(-8+6)^2$

$(-4)^2+(-2)^2$

$16+4$

$20 \neq 25$

$(-2,-6)$: $(-2-8)^2+(-6+6)^2=25$

$(-10)^2+0^2=$

$100 \neq 25$

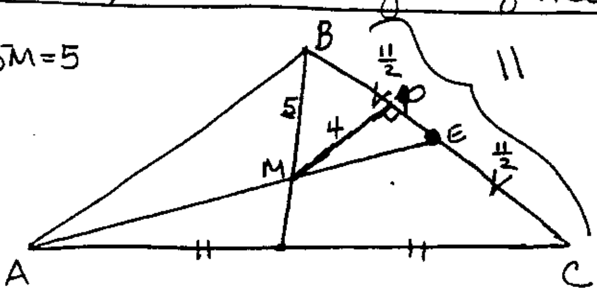
$(11,-2)$: $(11-8)^2+(-2+6)^2$

$3^2+4^2$

$9+16=25$ ✓

So $(11,-2)$ is a midpoint of a radius of the circle $\boxed{D}$

(19) $BM=5$




$BC=11$ let $E$ be the midpt of $BC$ then $BE=EC=\frac{11}{2}$

the distance from $M$ to $BC = 4$

$\Rightarrow$ let $P$ be the point of intersection of the $\perp$ $\Rightarrow$ by the pythagorean theorem $BP=3 \Rightarrow BE-BP=PE$ so $\frac{11}{2}-3=\frac{5}{2}=PE$ once again by the pythagorean theorem $ME=\sqrt{4^2+(\frac{5}{2})^2}=\sqrt{16+\frac{25}{4}}=\sqrt{\frac{64}{4}+\frac{25}{4}}=\sqrt{\frac{89}{4}}=\frac{\sqrt{89}}{2}$

$AM=2ME=2\left(\frac{\sqrt{89}}{2}\right)=\sqrt{89}$ $\boxed{D}$

**SOLUTION PAGE**

DIVISION: **ALPHA** TEST: **Analytic Geometry**

(20) Consider as vectors, then

$k(6,-3) = (4,-2)$

$6k = 4$ $\quad$ $-3k = -2$

$k = \frac{2}{3}$ $\quad$ $k = \frac{2}{3}$

$\boxed{D}$

Note:
From $(-1,2)$ to $(5,-1)$ is $(6,-3)$
From $(-1,2)$ to $(3,0)$ is $(4,-2)$

(21) Let the roots be $\frac{a}{r^2}$, $\frac{a}{r}$, $a$, $ar$, and $ar^2$. The sum of the roots $= \frac{a}{r^2} + \frac{a}{r} + a + ar + ar^2 = a\left(\frac{1}{r^2} + \frac{1}{r} + 1 + r + r^2\right) = 40$

the sum of the reciprocals $= \frac{1}{a}\left(\frac{1}{r^2} + \frac{1}{r} + 1 + r + r^2\right) = 10$

$\frac{1}{r^2} + \frac{1}{r} + 1 + r + r^2 = \frac{40}{a} = 10a$

$10a^2 = 40$

$a^2 = 4$

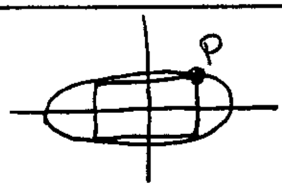
$a = \pm 2$

Since S is the negative of the product of the roots

$S = -a^5 = \pm 2^5 = \pm 32$

and $|S| = |\pm 32| = 32$ $\quad$ $\boxed{C}$

(22)




If a square is inscribed in an ellipse then the major and minor axes form 4 smaller squares. Consider the ellipse with center at the origin.

Point P's x and y coordinate would be equal. and substituting x for y in the equation

$p^2x^2 + q^2x^2 = c^2$

$(p^2 + q^2)x^2 = c^2$

$x^2 = \frac{c^2}{p^2+q^2}$

Since $p^2 + q^2 = c^2$

$x^2 = 1$

and area of large square is 4

$\boxed{A}$

(23) $\frac{(x+4)^2}{144} + \frac{(y-7)^2}{81} = 1$ $a = 12$, $b = 9$

the length of the latus rectum $= \frac{2b^2}{a} = \frac{2(9)^2}{12} = \frac{81}{6} = \frac{27}{2}$ $\boxed{E}$

(24) Amount of empty space = volume of the box − volume of cigars

$V_{Box} = 16r(30r) = 480r^2$

$V_{1^{st}way} = 120\pi r^2 \ell \Rightarrow$ Amount Empty $= 480r^2 - 120\pi r^2 \ell$

$V_{2nd\,way} = 131\pi r^2 \ell \Rightarrow$ Amount Empty $= 480r^2 - 131\pi r^2 \ell$

5 rows of 15 = 75
4 rows of 14 = 56
TOTAL = 131

DIFFERENCE

$(480r^2 - 120\pi r^2 \ell) - (480r^2 - 131\pi r^2 \ell)$
$= 11\pi r^2 \ell$ $\boxed{C}$

(25) See last page

(26) $r = 2 + 2\cos\theta$
$r = 2(1+\cos\theta)$

| $\theta$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ | $\frac{7\pi}{6}$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{2}$ | $\frac{11\pi}{6}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $1+\cos\theta$ | 2 | $\frac{2+\sqrt{3}}{2}$ | $\frac{3}{2}$ | 1 | $\frac{2-\sqrt{2}}{2}$ | 0 | .133 | $\frac{1}{2}$ | 1 | 1 | 1.866 |
| $r = 2(1+\cos\theta)$ | 4 | $2+\sqrt{3}$ | 3 | 2 | $2-\sqrt{2}$ .5857 | 0 | .2679 | 1 | 2 | 2 | 3.73 |

Cardioid
$r = 2(1+\cos\theta)$

180° rotation $\boxed{A}$

(27) $6y - x^2 = 36 + y^2 + 12x$

$-36 = x^2 + 12x + y^2 - 6y$

$-36 + 36 + 9 = x^2 + 12x + 36 + y^2 - 6y + 9$

$9 = (x+6)^2 + (y-3)^2$

Center $(-6, 3)$

$8x - x^2 = 23 - y$

$x^2 - 8x = y - 23$

$x^2 - 8x + 16 = y - 23 + 16$

$(x-4)^2 = y - 7$

Vertex $(4, 7)$

midpoint $\left(\frac{-6+4}{2}, \frac{3+7}{2}\right) = \left(\frac{-2}{2}, \frac{10}{2}\right) = (-1, 5)$ $\boxed{B}$

(28) $2x + 3y = 5$

$3y = -2x + 5$

$y = \frac{-2}{3}x + \frac{5}{3}$

$m_1 = -\frac{2}{3}$

$4x - 3y = 2$

$-3y = -4x + 2$

$y = \frac{4}{3}x + \frac{-2}{3}$

$m_2 = \frac{4}{3}$

$\tan\theta = \dfrac{\frac{4}{3} - \left(-\frac{2}{3}\right)}{1 + \left(\frac{-2}{3}\right)\left(\frac{4}{3}\right)}$

$= \dfrac{\frac{6}{3}}{1 + \frac{-8}{9}}$

$= \dfrac{2}{\frac{+1}{9}} = +18$ $\boxed{D}$

(29) $x^2 - 4x + y^2 + 2y + 3 = 0$

$x^2 - 4x + 4 + y^2 + 2y + 1 = -3$

$(x-2)^2 + (y+1)^2 = -3$ Center $(2, -1)$

$A = 2 - 1 = 1$ the $x$ intercepts are when $y = 0$ or

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3$ or $x = 1$

$\Rightarrow B = 3 + 1 = 4$

$A^2 + B^2 = 1^2 + 4^2 = 1 + 16 = 17$ $\boxed{B}$

(30) $2r = 3r\cos\theta + 6$

$2r - 3r\cos\theta = 6$

$r(2 - 3\cos\theta) = 6$

$r = \dfrac{6}{2 - 3\cos\theta}$

$r = \dfrac{6}{2\left(1 - \frac{3}{2}\cos\theta\right)}$ $\boxed{B}$

$e = \frac{3}{2}$

**SOLUTION PAGE**

DIVISION: **ALPHA** TEST: **Analytic Geometry**

25

The direction numbers are

for (2, 1, 4) ; (−1, 4, 1) & for (0, 5, 1) ; (3, −1, −2)

$$-1-2,\quad 4-1,\quad 1-4 \qquad\qquad 3-0,\quad -1-5,\quad -2-1$$

$$-3,\quad 3,\quad -3 \qquad\qquad 3,\quad -6,\quad -3$$

The direction of cosines for the two lines are

$$\frac{-3}{\sqrt{(-3)^2+3^2+(-3)^2}},\ \frac{3}{\sqrt{27}},\ \frac{-3}{\sqrt{27}} \qquad \frac{3}{\sqrt{3^2+(-6)^2+(-3)^2}},\ \frac{+6}{\sqrt{54}},\ \frac{-3}{\sqrt{54}}$$

$$\frac{-\sqrt{3}}{3},\ \frac{\sqrt{3}}{3},\ \frac{-\sqrt{3}}{3} \qquad \frac{\sqrt{6}}{6},\ \frac{+\sqrt{6}}{3},\ \frac{-\sqrt{6}}{6}$$

So $\cos\theta$ =

$$\left(\frac{-\sqrt{3}}{3}\right)\left(\frac{\sqrt{6}}{6}\right)+\left(\frac{\sqrt{3}}{3}\right)\left(\frac{+\sqrt{6}}{3}\right)+\left(-\frac{\sqrt{3}}{3}\right)\left(-\frac{\sqrt{6}}{6}\right)$$

$$-\frac{\sqrt{18}}{18}+\frac{+\sqrt{18}}{9}+\frac{\sqrt{18}}{18}=+\frac{\sqrt{18}}{9}=\frac{+3\sqrt{2}}{9}=+\frac{\sqrt{2}}{3}$$

$\boxed{C}$