

# MU ALPHA THETA CONVENTION 1991

## ANALYTIC GEOMETRY TOPIC TEST

1. What are the coordinates of the focus of the parabola whose equation is  $y^2 - 6y + 8x + 25 = 0$ ?  
 A.  $(-2, 3)$     B.  $(-2, 5)$     C.  $(-2, 0)$     D.  $(-4, 3)$   
 E. none of these
2. Find the slope of the line represented by the following parametric equations:  $x = 7 - 3t$  and  $y = 2 + 4t$ .  
 A.  $-\frac{4}{3}$     B.  $-\frac{3}{4}$     C.  $\frac{2}{7}$     D.  $\frac{7}{2}$     E. none of these
3. What is the distance from the point  $(5, -3, 4)$  to the plane whose equation is  $2x - 6y + 3z = 12$ ?  
 A. 1    B.  $\frac{16}{7}$     C.  $\frac{52}{7}$     D. 4    E. none of these
4. What is the tangent of the acute angles formed when the two lines whose equations are  $3x - 4y = 7$  and  $2x + 3y = 8$  intersect?  
 A.  $-\frac{17}{6}$     B.  $\frac{17}{6}$     C.  $\frac{1}{6}$     D.  $\frac{17}{18}$     E. none of these
5. What is the area, in square units, of the triangle whose vertices are the vertex and the endpoints of the latus rectum of the parabola whose equation is  $x^2 + 10x + 12y - 11 = 0$ ?  
 A. 18    B. 20    C. 36    D. 9    E. none of these
6. What are the asymptotes to the graph of  $y = \frac{x^2 - 4x + 3}{x^2 - 3x + 2}$ ?  
 A.  $x = 1$  and  $y = 2$  only    B.  $x = 1, x = 2,$  and  $x = 3$  only  
 C.  $x = 2$  and  $y = 1$  only    D.  $x = 1, x = 2,$  and  $y = 1$  only  
 E. none of these

7. What is the best description of the locus of the polar equation  $r = a \sec^2 \frac{\theta}{2}$ , and  $a > 0$ ?
- A. two lines    B. parabola    C. ellipse    D. limaçon  
E. none of these
8. A wheel of radius 2 feet 3 inches is rolling along the outside of a wheel with radius 2 feet 9 inches. Both wheels are in the same plane. If the initial point of contact is marked on each wheel, what is the least number of revolutions the smaller wheel will make before the marks coincide again?
- A. 33    B. 27    C. 11    D. 9    E. none of these
9. What is the area, in square units, of the trapezoid that can be formed by joining the points of intersection of the curves whose equations are  $x^2 + 4y^2 = 6x + 8y + 67$  and  $x^2 = 6x + 8y + 31$ ?
- A. 72    B. 144    C. 96    D. 36    E. none of these
10. What is the y-intercept of the line which is the extended common chord of the circles whose equations are  $x^2 + y^2 - 2x - 5y = 17$  and  $x^2 + y^2 + 4x + 7y = 9$ ?
- A.  $(0, -\frac{4}{3})$     B.  $(0, -\frac{2}{3})$     C.  $(0, -1)$     D.  $(0, \frac{4}{3})$   
E. none of these
11. Given that  $3x^2 + 8xy + 3y^2 - 10x - 30y + 20 = 0$  is not a degenerate conic, what is the shape of its graph?
- A. circle    B. ellipse    C. hyperbola    D. parabola  
E. none of these
12. Identical cannon balls are arranged in a pyramid shaped pile such that each horizontal layer of balls forms an equilateral triangle. If the bottom layer has 20 cannon balls on a side, the next layer has 19 balls on a side, etc., and the top layer has 1 ball, what is the total number of balls in the pile?
- A. 1540    B. 1490    C. 1450    D. 1250    E. none of these

13. What is the area, in square units, of the region enclosed by the graph of the parametric equations  $x=8\cos\theta$  and  $y=6\sin\theta$ ?
- A.  $96\pi$    B.  $48\pi$    C.  $12\pi$    D. 48   E. none of these
14. For what value(s) of  $k$  will the locus be imaginary for the equation  $x^2+4y^2-8y+k^2=0$ ?
- A.  $\pm 2$  only   B.  $-2 < k < 2$    C.  $k > 2$    D.  $k < -2$  or  $k > 2$   
E. none of these
15. Let  $(a,b)$  be the coordinates of the point of intersection of the graph of the oblique (slant) asymptote of  $f(x)$  with the graph of function  $f(x)$  where  $f(x) = \frac{(x-2)^3}{x^2}$ . What is the sum  $a+b$ ?
- A. oblique asymptote does not intersect  $f(x)$    B. 6   C.  $\frac{16}{9}$   
D.  $-\frac{14}{3}$    E. none of these
16. What is the equation of the line tangent to the circle  $x^2+y^2+2x-4y-5=0$  at the point  $P(2,1)$ ?
- A.  $3x+y=7$    B.  $x-3y=-1$    C.  $3x-y=5$    D.  $x+3y=5$   
E. none of these
17. The equation of one of the asymptotes to the graph of  $4x^2-y^2-2y-16x+19=0$  is
- A.  $y=2x$    B.  $2x-y=5$    C.  $x-2y=4$    D.  $y=4x-7$   
E. none of these
18. An arch is in the form of a semiellipse. The arch is 52 meters wide at the base and has a height of 20 meters at its highest point. How wide, in meters, is the arch at a height of 10 meters above the base?
- A. 40   B.  $26\sqrt{3}$    C.  $13\sqrt{3}$    D. 26   E. none of these

19. What is the area, in square units, of the smaller region enclosed by the graphs of  $3y = \sqrt{3}|x|$  and  $x^2 + y^2 = 6$ ?
- A.  $12\pi$    B.  $6\pi$    C.  $2\pi$    D.  $\pi$    E. none of these
20. Given points  $A(5,2)$ ,  $B(0,b)$ , and  $C(20,22)$ , what is  $b$ ,  $b > 0$ , so that  $AB + BC$  is a minimum?
- A. 3   B. 6   C. 10   D. 12   E. none of these
21. What is the area, in square units, of triangle  $ABC$  with vertices  $A(1,-1,1)$ ,  $B(2,2,2)$ , and  $C(4,-2,1)$ ?
- A. 5.5   B.  $\frac{9\sqrt{3}}{2}$    C.  $\frac{\sqrt{110}}{2}$    D. 6.25   E. none of these
22. The point  $P(2,5)$  bisects a chord of the circle whose equation is  $x^2 + y^2 - 6x - 4y = 36$ . Let  $(a,0)$  be the  $x$ -intercept of this chord extended. What is  $a$ ?
- A. -13   B. -14   C. -15   D. -16   E. none of these
23. There are four congruent circles tangent externally to a given circle with radius equal to five. The four congruent circles are placed around the given circle so as to be tangent to each other successively. What is the length of the radius of each of the four congruent circles? All five circles are in the same plane.
- A. 8   B. 10   C.  $5(2\sqrt{2}+1)$    D.  $5(\sqrt{2}+1)$   
E. none of these
24. What is the area, in square units, of the region enclosed by the directrices and the tangents at the endpoints of the minor axis of the ellipse whose equation is  $9x^2 + 25y^2 - 36x + 50y = 164$ ?
- A.  $\frac{75}{2}$    B. 60   C. 72   D. 75   E. none of these

25. What is the volume, in cubic units, of a tetrahedron whose vertices are  $A(3,1,0)$ ,  $B(1,4,0)$ ,  $C(5,5,0)$  and  $D(3,1,6)$ ?

- A. 14    B. 24    C. 42    D. 84    E. none of these