Analytic Geometry Topic Test Key:

1. D
2. A
3. D
4. B
5. A
6. C
7. B
8. C
9. A
10. B
11. C
12. A
13. B
14. D
15. D
16. C
17. B
18. B
19. C
20. B
21. C
22. A
23. D
24. D
25. A

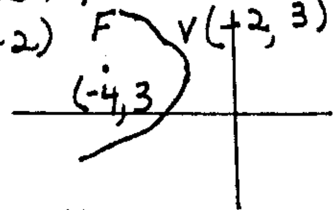
Analytic Geometry Topic Test Solutions

① $y^2 - 6y + 9 = -8x - 25 + 9$ ans D

$(y-3)^2 = -8(x+2)$

$4p = -8$

$p = -2$



② By inspection $-\frac{4}{3}$ ans. A

alt. sol.: $\begin{cases} x = 7 - 3t \\ y = 2 + 4t \end{cases}$

$\begin{cases} 4x = 28 - 12t \\ 3y = 6 + 12t \end{cases}$

$4x + 3y = 34$

$y = -\frac{4x}{3} + \frac{34}{3}$

$m = -\frac{4}{3}$

③ ans D

$$d = \frac{|Ax^* + By^* + Cz^* + D|}{\sqrt{A^2 + B^2 + C^2}}$$

$$d = \frac{|2 \cdot 5 + (-6)(-3) + 3(4) + (-12)|}{\sqrt{2^2 + (-6)^2 + 3^2}}$$

$$d = \frac{28}{7} = 4$$

④ Let θ = acute angle formed ans B

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ absolute value since θ is acute

$m_1 = \frac{3}{4} \quad m_2 = -\frac{2}{3}$

$$\tan\theta = \frac{\frac{3}{4} - (-\frac{2}{3})}{1 + \frac{3}{4}(-\frac{2}{3})}$$

$$= \frac{9+8}{12-6} = \frac{17}{6}$$

⑤ $\cancel{x^2+10x+25=-12y+11+25}$

$\cancel{(x+5)^2=-12(y-3)}$

~~latus rectum = 12~~

$\cancel{h=3}$

$\cancel{A=\tfrac{1}{2}bh}$

$\cancel{A=\tfrac{1}{2}(12)(3)=18}$

~~yes~~

ans A

⑥ $y=\dfrac{(x-3)(x-1)}{(x-1)(x-2)}$

ans. C

vert. asym.: $x=2$

horiz. asym: $\lim\limits_{x\to\infty}\dfrac{x-3}{x-2}=1$

$\therefore y=1$

no slant asymptote

$x=2$ and $y=1$ only

⑦ $r=a\dfrac{1}{\cos^2\frac{\theta}{2}}$

ans. B

$r=a\dfrac{1}{\frac{1}{2}(\cos\theta+1)}$

$r=\dfrac{2a}{\cos\theta+1}$

by inspection $e=1$ $\therefore$ parabola

or can change to rectangular coordinates: $y^2=-4a(x-a)$

⑧ Both wheels must travel same distance

ans. C

$\therefore C_1=C_2$

$33\cdot(2\pi)N_1=27(2\pi)N_2$

$\dfrac{N_1}{N_2}=\dfrac{27}{33}$

$\dfrac{N_1}{N_2}=\dfrac{9}{11}$

$\therefore$ Smaller wheel makes 11 rev.

⑨ $6x+8y+31+4y^2=6x+8y+67$

$4y^2=36$

$y=\pm 3$

$x^2=6x+55$

$x^2-6x-55=0$

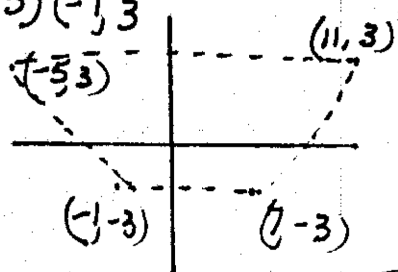
$(x-11)(x+5)=0$ $(-5,3)(11,3)$ ans A

when $y=-3$

$x^2=6x+7$

$x^2-6x-7=0$ $(7,-3)(-1,3$

$(x-7)(x+1)=0$



$A=\frac{1}{2}h(b_1+b_2)$

$A=\frac{1}{2}6(16+8)$

$=72$

ans. B

⑩ ~~$x^2+y^2-2x-5y=17$~~

yes

~~$x^2+y^2+4x+7y=9$~~

~~$6x+12y=-8$~~

~~y int $=-\frac{2}{3}$~~

⑪ use discriminant ans C

$B^2-4AC>0 \rightarrow$ hyperbola

$64-4(3)(3)=28$

$\therefore$ hyperbola

~~$9(x^2+2x\ \ )+25(y^2-4y\ \ )=116$~~ $+9$ $+100$

~~$9(x+1)^2+25(y-2)^2=225$~~

~~$\frac{(x+1)^2}{25}+\frac{(y-2)^2}{9}=1$~~ major $=10$ minor $=6$ ~~$10^2+6^2=136$~~

(12) No. of balls $= \dfrac{n(n+1)(n+2)}{1\cdot 2\cdot 3}$

$= \dfrac{\overset{10}{\cancel{20}}\cdot\overset{7}{\cancel{21}}\cdot 22}{1\cdot\cancel{2}\cdot\cancel{3}} = 1540$ ans A

alt. solution - count them:

$1+3+6+10+15+21+\ldots+210 = 1540$

(13) $\begin{cases} \frac{x}{8} = \cos\theta \\ \frac{y}{6} = \sin\theta \end{cases}$ ans B

$\dfrac{x^2}{64} + \dfrac{y^2}{36} = \cos^2\theta + \sin^2\theta$

$\dfrac{x^2}{64} + \dfrac{y^2}{36} = 1$

$\therefore$ ellipse $A = \pi a b$

$= \pi\cdot 8\cdot 6$

$= 48\pi$

(14) $x^2 + 4y^2 - 8y^2 = -k^2$ ans D

$x^2 + 4(y^2 - 2y + 1) = -k^2 + 4$

$-k^2 + 4 < 0$

$k^2 > 4$

$|k| > 2$

$k > 2$ or $k < -2$

(15) $f(x) = \dfrac{x^3 - 6x^2 + 12x - 8}{x^2}$ ans D

$f(x) = x - 6 + \dfrac{12x - 8}{x^2}$

set $12x - 8 = 0$

$x = \frac{2}{3}$

$y = -\frac{16}{3}$

$x + y = \frac{-14}{3}$

(16) $C(-1,2) \quad P(2,1)$

$m_r = \frac{2-1}{-1-2} = -\frac{1}{3}$ ans C

$m_t = \frac{3}{1} \quad P(2,1)$

$\therefore 3x - y = 5$

(17) $4x^2 - 16x - (y^2 + 2y + \quad) = -19$

$4(x-2)^2 - (y+1)^2 = -4$ ans B

$\frac{(y+1)^2}{4} - \frac{(x-2)^2}{1} = 1$

asymptotes

$\left(\frac{y+1}{2}\right)^2 - (x-2)^2 = 0$

$\frac{y+1}{2} = \pm(x-2)$

$y = 2x - 5$ or $y = -2x + 3$

$y = 2x - 5$

(18) $\frac{x^2}{26^2} + \frac{y^2}{20^2} = 1$ ans. B

Find x when $y = 10$

$\frac{x^2}{26^2} + \frac{100}{400} = 1$

$\frac{x^2}{26^2} = \frac{3}{4}$

$x = \frac{26\sqrt{3}}{2}$

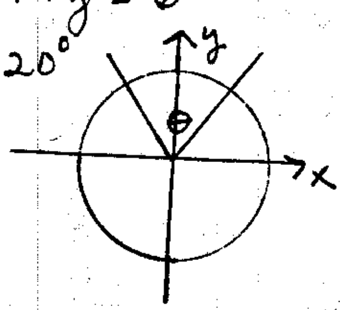
width $= 2x = 26\sqrt{3}$

(19) $y = \frac{\sqrt{3}}{3}|x| \quad x^2 + y^2 = 6$ ans C

$m = \frac{\sqrt{3}}{3} \therefore \theta = 120^\circ$

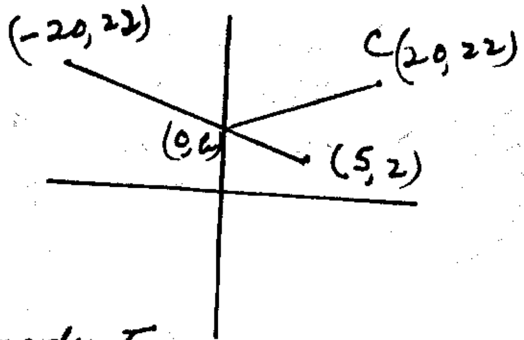
$A = \frac{1}{3} \cdot \pi \cdot 6$

$= 2\pi$



(20) a st. line is the shortest distance between 2 pts.

Use slopes



$$\frac{22-b}{-20} = \frac{22-2}{-20-5}$$

$b = 6$

ans B

(21) Use cross products

$\overrightarrow{AB} = (1,3,1)$ $\overrightarrow{AC} = (3,-1,0)$

$\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 3 & -1 & 0 \end{vmatrix} = \frac{1}{2}\left|(1, 3, -10)\right|$

$A = \frac{1}{2}\sqrt{1+9+100} = \frac{1}{2}\sqrt{110}$

ans C

alt. sol. Use Hero's formula

$d_{AB} = \sqrt{11}$ $d_{AC} = \sqrt{10}$ $d_{BC} = \sqrt{21}$

$\frac{1}{2}P = s = \frac{\sqrt{11}+\sqrt{10}+\sqrt{21}}{2}$.

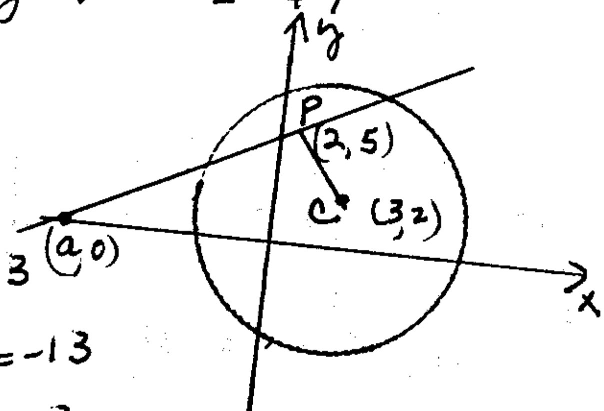
$$A = \sqrt{\left(\frac{\sqrt{11}+\sqrt{10}+\sqrt{21}}{2}\right)\left(\frac{\sqrt{11}+\sqrt{10}+\sqrt{21}-2\sqrt{11}}{2}\right)\left(\frac{\sqrt{11}+\sqrt{10}+\sqrt{21}-2\sqrt{10}}{2}\right)\left(\frac{\sqrt{11}+\sqrt{10}+\sqrt{21}-2\sqrt{21}}{2}\right)}$$

$$A = \sqrt{\frac{440}{16}} = \sqrt{\frac{110}{4}} = \frac{1}{2}\sqrt{110}$$

(22) $x^2-6x+9+y^2-4y+4 = 36+9+4$

ans A

$(x-3)^2+(y-2)^2 = 49$



$m_{CP} = -\frac{3}{1}$

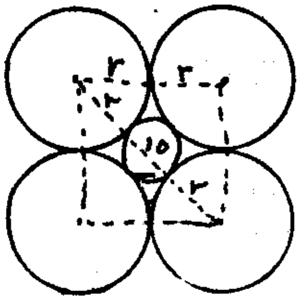
$m_{chord} = \frac{1}{3}$

eq.: $x-3y=-13$

x int $=-13$

$a=-13$

(23)



side of sq $= 2r$

diagonal of sq $= 2r\sqrt{2}$ ans D

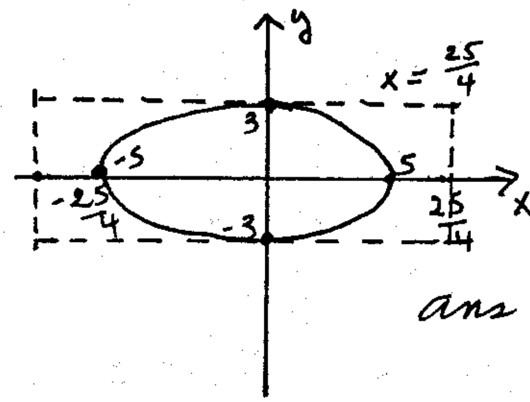
diagonal of sq $= 10 + 2r$

$\therefore 2r\sqrt{2} = 10 + 2r$

$r = \dfrac{5}{\sqrt{2}-1}$

$r = 5(\sqrt{2}+1)$

(24) $\dfrac{(x-2)^2}{25} + \dfrac{(y+1)^2}{9} = 1$



ans D

$a = 5,\ b = 3,\ c = 4$

directrices $= \pm \dfrac{a}{e}$

$e = \dfrac{4}{5}$

directrices $x = \pm \dfrac{25}{4}$

area $= (2x)\,6$

$= \dfrac{25}{2} \cdot 6 = 75$

(25) Solution: Use determinant

Ans A

$$\frac{1}{6}\begin{vmatrix} 3 & 1 & 0 & 1 \\ 1 & 4 & 0 & 1 \\ 5 & 5 & 0 & 1 \\ 3 & 1 & 6 & 1 \end{vmatrix} = \frac{1}{6}\begin{vmatrix} 3 & 1 & 0 & 1 \\ -2 & 3 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 6 & 0 \end{vmatrix} =$$

$$\frac{1}{6}(6)\begin{vmatrix} 3 & 1 & 1 \\ -2 & 3 & 0 \\ 2 & 4 & 0 \end{vmatrix} = -1\begin{vmatrix} -2 & 3 \\ 2 & 4 \end{vmatrix}$$

$$-1(-8-6) = 14$$

could use Hero's formula to calculate area of base and then use distance from pt. to plane to get height.