

Mu Alpha Theta National Convention: Seattle, 1997
Calculus Trigonometry Topic Test

8. A plane is flying at 270 kilometers (km) an hour at an altitude of 6 km, and a man is watching it from directly underneath its flight path. How long, to the nearest second, after the man had to look up at 8 degrees must the man first look up at 48 degrees?
- (A) 473 (B) 488 (C) 503 (D) 512 (E) NOTA
9. A woman is standing at the foot of a hill which rises at 17 degrees. There is a vertical tower built 700 meters (measured along the slope of the hill) from her. The woman must look up at an angle of 34 degrees to see the top of the tower. What is the tower's height, to the nearest meter?
- (A) 238 (B) 242 (C) 247 (D) 253 (E) NOTA
10. If $\sin x = .7$, and x is between $\frac{\pi}{2}$ and π , what is the value of $\cos\left(\frac{x}{2}\right)$, to the nearest hundredth?
- (A) .37 (B) .38 (C) .92 (D) .93 (E) NOTA
11. In triangle ABC, $a=7$, $b=4$, and $A=43^\circ$. What is the measure of C to the nearest degree?
- (A) 108 (B) 110 (C) 112 (D) 114 (E) NOTA
12. Simplify $\tan^2 x + 1$
- (A) $1 - \csc x$ (B) $\csc x$ (C) $2 \sin^2 x$ (D) $\sec^2 x$ (E) NOTA
13. A man is trying to row across a half kilometer wide river and arrive at the other side exactly opposite where he started. If he can row at 15 km/hr in still water, and the river's current is 6 km/hr, how long will it take him to reach the other side, to the nearest second?
- (A) 128 (B) 131 (C) 135 (D) 140 (E) NOTA
14. Which of the following is equal to $\frac{\sin^2 x}{\cos^3 x}$?
- (A) $\cot x \sin x$ (B) $\sec x \tan^2 x$ (C) $\sin x \tan^2 x$ (D) $\cos^3 x \sin^2 x$ (E) NOTA

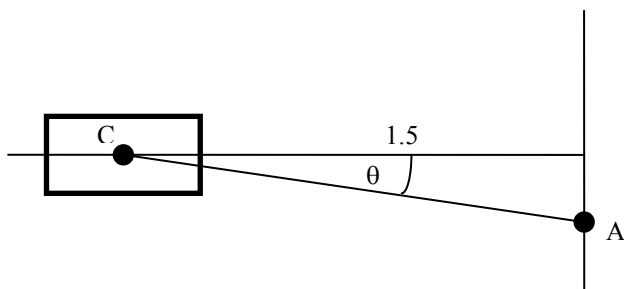
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15. What is the tangent of the angle in the third quadrant which has a sine of $-\frac{5}{13}$
- (A) $\frac{5}{12}$ (B) $\frac{12}{5}$ (C) $-\frac{5}{12}$ (D) $-\frac{12}{5}$ (E) NOTA

16. For which of the following values of x ($0^\circ \leq x < 360^\circ$) does $2 \sin^2 3x - \sin 3x - 1 = 0$?
- I. 30°
 II. $110^\circ, 230^\circ$
 III. $150^\circ, 270^\circ$
 IV. $180^\circ, 300^\circ$
 V. $230^\circ, 350^\circ$

- (A) I, III (B) II, IV, & V (C) I, III, & V (D) I, II, III & V (E) NOTA

17. A 30 cm by 50 cm Mars rover will detect an obstacle when it is 1.5 meters ahead of its center, and assume it is the leading edge of a circular rock of radius 80 cm. The rover then pivots the minimum angle about its center (C) that will allow it to pass such a rock with 10 cm of clearance. If obstacle A is detected $\theta = 18^\circ$ away from straight ahead, how many degrees will the rover's navigation algorithm cause it to pivot, to the nearest hundredth of a degree?



- (A) 20.96 (B) 22.12 (C) 28.43 (D) 29.77 (E) NOTA

18. Evaluate: $\sum_0^{20} \cos\left(\frac{\pi}{7} + \frac{2\pi i}{21}\right)$

- (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) NOTA

19. What is the largest possible value of the sum of the sines of the angles in a triangle?

- (A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{5}{2}$ (C) $2\sqrt{3}$ (D) 3 (E) NOTA

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20. Determine $y\left(\frac{\pi}{2}\right)$ if $\frac{dy}{dx} = \sin^2 x$ and $y(0) = 1$.
- (A) $\frac{\pi}{3} + 1$ (B) $\frac{\pi}{4} + 1$ (C) $\frac{\pi + 2}{3}$ (D) $\frac{\pi}{4} + \frac{1}{2}$ (E) NOTA
21. At what rate, to the nearest hundredth of a mm/s, is the shadow of a 2 m tall man lengthening when the sun is 15° above sunset? Assume the sun is dropping at $\frac{2\pi}{24 \cdot 3600}$ rad/s.
- (A) 1.89 (B) 1.94 (C) 2.03 (D) 2.10 (E) NOTA
22. Train A is 5 km west of Chicago (C) heading east at 80 kmph, while Train B is 12 km north of Chicago heading out at 50 kmph. At what rate is angle CAB changing?
- (A) $\frac{1023}{169}$ (B) $\frac{1097}{169}$ (C) $\frac{1143}{169}$ (D) $\frac{1210}{169}$ (E) NOTA
23. $y = \cos^{-1} x$. Determine $\frac{dy}{dx}$.
- (A) $\frac{1}{\sqrt{1+x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{1+x^2}$ (D) $\frac{1}{1-x^2}$ (E) NOTA
24. $\frac{dy}{dx} = \frac{2 \sin x}{\cos^2 x}$. Determine $y(x)$ if $y\left(\frac{\pi}{3}\right) = 1$.
- (A) $\csc^2 x - \frac{1}{3}$ (B) $\ln|\tan^2 x| + \ln \frac{e}{3}$
(C) $2\sqrt{1 - \sin^2 x}$ (D) $2 \sec x - 3$ (E) NOTA
25. What is the derivative of $y = \tan^3 4x$?
- (A) $12 \tan^2 4x \sec 4x$ (B) $-3 \tan^2 4x \sec 4x$
(C) $-12 \tan^3 4x \sec 4x$ (D) $4 \tan^3 4x \sec 4x$ (E) NOTA

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26. Determine $y\left(\frac{2\pi}{3}\right)$ if $y\left(\frac{\pi}{3}\right) = 1$ and $\frac{dy}{dx} = \sec x$.

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1 (E) NOTA

27. Determine $y(x)$ if $\frac{dy}{dx} = \cos^3 x$.

- (A) $\sin x - \frac{1}{3} \sin^3 x$ (B) $\frac{1}{4} \cos^4 x$
(C) $3 \sin x + \sin^2 x$ (D) $\frac{\cos^4 x}{4 \sin x}$ (E) NOTA

28. Determine $\left.\frac{dy}{dx}\right|_{y=2}$ if $\tan xy = 1$.

- (A) $-\frac{16}{\pi}$ (B) $\frac{4}{\pi}$ (C) $-\frac{\pi}{16}$ (D) $-\frac{\pi}{4}$ (E) NOTA

29. If $\sin A = \frac{8}{17}$ for A in the second quadrant and $\cos B = \frac{5}{13}$ for B in the fourth quadrant, what is the value of $\sin(A - B)$?

- (A) $-\frac{120}{221}$ (B) $-\frac{140}{221}$ (C) $-\frac{170}{221}$ (D) $-\frac{190}{221}$ (E) NOTA

30. Determine the solution set of $\sin^6 x + \cos^6 x = 1$. Note: in each answer n is an element of the integers.

- (A) $x = \frac{n\pi}{4}$ (B) $x = \frac{(2n+1)\pi}{4}$
(C) $x = \frac{n\pi}{3}$ (D) $x = \frac{n\pi}{2}$ (E) NOTA