Mu Alpha Theta National Convention 2004
Statistics Solutions

1. **B.** $z=\frac{x-\mu}{\sigma}$; $2.98=\frac{x-75.2}{6.2}$; $x=93.68$

2. **C.** $s=\frac{99-75}{12}=2$; $s=\pm 2$; $1-\left(\frac{1}{2}\right)^2=.75$; 75% of the values will be within two standard deviations of the mean so 25% will be less than 2 or more than 2 standard deviations of the mean; 25% (600) = 150.

3. **A.** $Q_3-Q_1=34-14=20$

4. **D.** Find the z-score for the 90th percentile (0.90) on a Standard Normal Probabilities Table which is z = 1.28. (Use the formula $z=\frac{x-\mu}{\frac{\sigma}{\sqrt{n}}}$ with $\mu=100, n=16, \sigma=2$.) **OR** on the TI-83 calculator Invnorm(0.90, 100, $2/\sqrt{16}$) = 100.64

5. **A.** 70(.15) + 80(.20) + 64(.20) + 77(.20) + 96(.25) = 78.7

6. **B.** $CV=\frac{s}{\bar{x}}\cdot 100\%=\frac{4.06}{28.19}\cdot 100\%=14.4\%$

7. **A.** P(Sophomore or Male)
$= $ P(Sophomore) + P(Male) – P(Sophomore and Male)
$=\frac{15}{26}+\frac{18}{26}-\frac{10}{26}=\frac{23}{26}$.

8. **B.** P(from Box A|Red) $=\frac{P(Box A and Red)}{P(Red)}=\frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{6}}=\frac{6}{11}$.

9. **D.**

Basketball	500(.24) = 120
Football	500(.38) = 190
Hockey	500(.16) = 80
Soccer	500(.18) = 90
Other	500(.04) = 20

10. **C.** P(A and B) = P(A) P(B|A) = (.40)(.65) = .26

11. **B.** mutually exclusive events have no outcomes in common therefore P(A and B) = 0.

12. **E.** $\sigma^2=\Sigma(x^2\cdot p(x))-[\Sigma x\cdot p(x)]^2=2.9-(1.32)^2=1.1576$

13. **B.** P(one good and one bad or one bad and one good)
$=\left(\frac{10}{12}\right)\left(\frac{2}{11}\right)+\left(\frac{2}{11}\right)\left(\frac{10}{12}\right)=\frac{10}{33}=.3030$

14. **B**

15. **D.** margin of error ≥ $z^* \frac{\sigma}{\sqrt{n}}$; $10 \geq 1.96\frac{75}{\sqrt{n}}$; $n \geq 216.09$

16. **B.** The Z-Score with 3.62% of the observations falling above it = 1.80.

$$z = \frac{x-\mu}{\sigma}; \quad 1.80 = \frac{572-500}{\sigma}; \quad \sigma = 40$$

17. **A.** Substituting the value x = 3 into the equation of the least-squares regression line results in $\hat{y} = 1.3 + 0.27(3) = 1.3 + 0.81 = 2.11$. The value of the residual is observed – predicted = 2-2.11 = -0.11

18. **A.** margin of error = $z^*\left(\sqrt{\frac{(p)(q)}{n}}\right) = \pm 2.326\left(\sqrt{\frac{(.5)(.5)}{2172}}\right) = 0.025$

19. **B.** Type II error is the mistake of failing to reject the null hypothesis when it is false. A Type I error is the mistake of rejecting the null hypothesis when it is true.

20. **B.** When the confidence level goes from 95% to 99% the confidence interval must be wider. II is the only interval which is larger.

21. **C.** Choice A is correct; pooled estimate of the population variance is

$$\frac{x_1 + x_2}{n_1 + n_2} = \frac{35+25}{60+70} = \frac{60}{130} = \frac{6}{13}.$$

Choice B is correct; standard error is

$$\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = \sqrt{\frac{\left(\frac{35}{60}\right)\left(\frac{25}{60}\right)}{60} + \frac{\left(\frac{25}{70}\right)\left(\frac{45}{70}\right)}{70}} = .086$$

Choice C is incorrect. The test statistic is $z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$.

$$\hat{p}_1 = \frac{35}{60}; \hat{p}_2 = \frac{25}{70}; \hat{p} = \frac{n_1}{n_1 + n_2} = \frac{60}{60+70} = \frac{60}{130}$$

$z = 2.58$

Choice D is correct. The p-value is 0.0099 which is less than .05 so the decision is to reject H_0.

22. **B.** $z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.49 - .46}{\sqrt{\frac{(.46)(.54)}{1000}}} = 1.90$

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23. **C.** $\bar{x}_1 - \bar{x}_2 \pm z^* \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = 25.50 - 22.50 \pm 2.58\sqrt{\frac{4.75^2}{32} + \frac{2.83^2}{35}} = 3 \pm 2.49$

24. **C.** The coefficient of determination, r^2, is the fraction of the variation in the values of $\hat{y}$ that is explained by the least square regression of y on x. The correlation is r = 0.597, we square it to get our answer $(0.597)^2 = 0.356$.

25. **C.** Find x_o so that $p(x \le x_o) = 2\% = 0.02$. First, find z_o so that $p(z \le z_o) = 0.12$. $p(z \le -2.05) = 0.02$. $x_o = \mu + z_o\sigma = 8 + (-2.05)(1.2) = 5.54$.

26. **D.** $\hat{y} = 12.3154 + 0.7526x$; $\hat{y}(58) = 55.9$.

27. **D.**

35	50	20	27	76	33	31	5	49	64	63	12
13	4	30	45	29	16	71	11	89	40	81	77
46											

There are eleven values between 20 and 49 so eleven sophomores were selected.

28. **C.** $\hat{y} = 243.7 - 0.037(450) + 7.35(5) = 263.8$

29. **B.** $n = \left(\frac{z^*(\sigma)}{E}\right)^2 = \left(\frac{(1.645)(0.035)}{0.02}\right)^2 = 8.29$. The minimum must be larger than 8.29.

30. **A.** The interquartile range (IQR) is 49-27 = 22. To determine an outlier compute 1.5 x IQR and add this value to Q_3 and subtract it from Q_1. Any data that falls outside the range of these two values is an outlier.

$$Q_3 + 1.5(22) = 49 + 33 = 82$$
$$Q_1 - 1.5(22) = 27 - 33 = -6$$

Since all the values in A fall above 82, they are outliers.

Tie Breaker 1: Ans: **0.0823**

The test accepts H_o when $-1.96 \le \frac{\bar{x} - 2}{\frac{.01}{\sqrt{5}}} \le 1.96$ or $1.9912 \le \bar{x} \le 2.0088$.

The probability of a Type II error is $p(1.9912 \le \bar{x} \le 2.0088)$ if $\mu = 2.015$ and the standard error is $\frac{.01}{\sqrt{5}}$.

$$p\left(\frac{1.9912 - 2.015}{\frac{.01}{\sqrt{5}}} \le \frac{\bar{x} - 2.015}{\frac{.01}{\sqrt{5}}} \le \frac{2.0088 - 2.015}{\frac{.01}{\sqrt{5}}}\right) = p(-5.32 \le z \le -1.39) = \mathbf{.0823}$$

Tie Breaker 2: Ans: **162.91**

$$X^2 = \sum \frac{(observedcount - \exp ectedcount)^2}{\exp ectedcount}; \quad X^2 = 88.57 + 13.80 + 7.56 + 6.85 + 46.13 = \mathbf{162.91}$$

Tie Breaker 3: Ans: **(2.92 to 5.11)**

95% CI for β: n = 36, so df = 34. Using technology or interpolation, critical t = 2.032

$b \pm t_{\alpha/2} s_{b_1} = 4.0162 \pm 2.032(.5393) =$ **(2.92 to 5.11)**