

Mu Alpha Theta 2004 State Bowl Questions - Mu

Round 1

1. Find all value(s) of t , such that $tt_b = 106_7$.
2. The length of a rectangle is increasing at 3 times the rate that its width is increasing. The diagonal of the rectangle is increasing how many times faster than the width when the length is 12 feet and the width is 6 feet?

Round 2

3. Let $f(x)$ be a linear function such that $\int_0^2 f(x) dx = 10$. Find the sum of the slope and the y-intercept of the graph of f .
4. Find the average value of $f(x) = \sin x$ on $[0, \frac{\pi}{2}]$.

Round 3

5. $\int x(x+3) dx =$
6. If the function $f(x)$ is continuous and differentiable and $f(x) = \begin{cases} ax^3 - 6x; & \text{if } x \leq 1 \\ bx^2 + 4; & \text{if } x > 1 \end{cases}$ then $a =$

Round 4

7. Let P = number of grams of the radioactive element petetonium. The rate of change in P is directly proportional to the amount present. At $t = 1$, there are 16 grams of P present. At $t = 5$, there are 4 grams present. At what time t will there be exactly 1 gram of P present?
8. A rectangle is to be inscribed in the ellipse $\frac{x^2}{100} + \frac{y^2}{50} = 1$ with sides parallel to the coordinate axes. What is the maximum area of the rectangle?

Round 5

9. If $y \frac{dy}{dx} = (4 + y^2)x^2$, and $y(0) = 12$, find y' .
10. If $\cos(9x) - \cos(7x) = 0$, find the number of solutions for x , where $0 < x < \frac{\pi}{2}$.

Round 6

11. The radius of convergence of $\sum_{n=1}^{\infty} \frac{a^n}{(x+2)^n}$; $a > 0$ is what?

12. Consider the set of all possible four-digit numbers consisting only of the digits 3, 4, 5, 6, and 7 with no repetition of the digits in a given number. Find the sum of all such numbers.

Round 7

13. Find $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$

14. Find the length of the arc of $y = 2\left(x - \frac{1}{9}\right)^{\frac{3}{2}}$ from $x = 4$ to $x = 16$.

Round 8

15. Find the area enclosed by the set of points defined by $|x| + |y| + |x+y| = 2$.

16. A slice of pie in the form of a sector of a circle is to have a perimeter of 18 inches. What should be the diameter of the circle in order to make the piece of pie largest?

Round 9

17. Evaluate $\int_1^e (\ln x)^2 dx$.

18. Given that $g(x)$ is an even function and that $\int_0^3 g(x) dx = 5$, evaluate $\int_{-3}^3 (4g(x) + 5) dx$.

Round 10

19. Evaluate $\lim_{y \rightarrow 0} \frac{\sin y - \tan y}{3y^3}$

20. Find the value of b , greater than 1, such that the average value of $f(x) = x^2$ on the interval $[1, b]$ equals 7.