2004 National Mu Alpha Theta Convention
Mu Division–Sequences and Series Topic Test

1. **C** The sequence decreases by 6 every 2 terms.

2. **B** The sequence triples every other term, so our answer is $(-1)(3^3) = -27$.

3. **D** We have $(3a - b) - (a + 2b) = (4a + 7b) - (3a - b)$, from which we deduce $a = 11b$, or $a/b = 11$.

4. **D** Factoring a 2 out of the bottom yields the top, so our sum is $1/2$.

5. **B** List out the terms and add them.

6. **B** Call the second term ar and the eighth ar^7. The geometric mean of these is $\sqrt{a^2r^8} = ar^4$, which is the fifth term of the sequence.

7. **B** Our sum is $1/(1 - i/2) = (4 + 2i)/5$.

8. **A** Only (A) matches the first term and the last term and has the proper common difference.

9. **D** Each term is of the form $3(2^n)$. The largest such that is less than 2000 is $3(2^9)$, so there are 10 terms less than 2000.

10. **C** First we find the c's, then the b's, then finally the a's. We get 1,2,4,7,11 for the first 5 c's; 1,2,4,8,15 for the b's; $1, 2, 4, 8, 16, 31$ for the a's.

11. **A** The series is $1/k$ times the harmonic series $1 + 1/2 + 1/3 + \cdots$. Therefore it diverges for all positive k.

12. **B** The described sequence must have every term the same. These terms could be all positive or all negative or all 0. Hence, the first 2 statements could be true.

13. **B** Any linear combination of arithmetic sequences is an arithmetic sequence. The product of two arithmetic sequences is not necessarily arithmetic.

14. **A** Solving $1/x - 1/2 = 3/2 - 1/x$ gives us $x = 1$.

15. **A** The sum of the first k squares is $(k)(k+1)(2k+1)/6$. The smallest positive k for which this is divisble by 11 is $k = 5$.

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16. D We could select a, b, and c to match any three terms of any arithmetic sequence. Hence, the desired quantity could take on many values.

17. C When $n = 6$, the term is 0; hence, the whole product is 0.

18. C Add the two geometric series a_n and b_n separately. The first gives 3/2 and the second gives 4/5. Thus, we have $3/2 - 3(4/5) = -9/10$ as our sum.

19. D By the integral test, this converges only for $p > 3/2$.

20. D We can write our equation as $\log_N 2 + \log_N 3 + \log_N 4 + \cdots + \log_N 9 = \log_N k$, from which $k = 9!$ follows.

21. B Let the expression equal $x - 1$; we have $x - 1 = 1/(1 + 1/x)$, from which $x - 1 = 1 \pm \sqrt{3}$. Clearly $x - 1$ is positive, so we have $x = \sqrt{3}$.

22. A We can factor out the j and find that our sum is the product of two arithmetic series, with sum $(15)(21) = 315$.

23. A Given that $e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$, we can simply put $3x$ in place of x to get our answer.

24. C The definition forces every other term to form an arithmetic sequence. This holds for both the terms in odd places and those in the evens. Nothing relates the odd terms to the evens, so the full sequence need not be arithmetic.

25. A Grouping the '9' terms and the '8' terms we have $(2^9 - 1) - 2^8 = 255$.

26. B In general, a_k for $1/(1-x)^n$ is $\binom{k+n-1}{n-1}$, which we can see by multiplying the infinite geometric series $1/(1-x) = 1 + x + x^2 + x^3 + \ldots$ the appropriate number of times.

27. B The fourth derivative of $f(x)$ is 0 at $x = 0$, so our coefficient is 0.

28. A In general, $\binom{n}{0}^2 + \ldots + \binom{n}{n}^2 = \binom{2n}{n}$. There are many ways to see this. Try equating coefficients of x^n in $(x+y)^n(x+y)^n$ and $(x+y)^{2n}$.

29. C Solving the characteristic equation of the recursion gives $x^2 - 2x - 1 = 0$, or $x = (1 \pm \sqrt{2})$. As n grows large, only the $1 + \sqrt{2}$ term matters, as the other vanishes. Hence, the ratio of successive terms tends towards $1 + \sqrt{2}$.

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30. **A** We can show that $2(\sqrt{n+1}-\sqrt{n}) < \frac{1}{\sqrt{n}} < 2(\sqrt{n}-\sqrt{n-1})$. Applying this to each term in the sum and adding the resulting inequalities gives us $2\sqrt{1000001}-2 < \sum < 2\sqrt{1000000}-0$, from which we deduce that our sum is between 1998 and 2000. If we instead write $1 \leq 1$ for the first term and $\frac{1}{\sqrt{n}} < 2(\sqrt{n}-\sqrt{n-1})$ for the rest and add we have $\sum < 1+2(\sqrt{1000000}-\sqrt{1})$, from which we deduce that our sum is less than 1999. Hence our sum has integer part 1998.

T1. **11527201** We can prove that $1^3+3^3+\cdots+(2n-1)^3 = n^2(2n^2-1)$. For $n=47$, we have our answer.

T2. **19958399** Add 1 to the expression and factor like mad to get

$$(1+2)(1+3)(1+4)\cdots(1+10).$$

Multiply that out, sutract 1, and you're done!