Sequences and Series - Mu Level
2000 Mu Alpha Theta National Convention

For all questions , answer E. "NOTA" means none of the above

1. $\sum_{n=-100}^{100} 5n + 3 =$

A. 0 B. 3 C. 600 D. 603 E. NOTA

2. As n tends to infinity, the sequence $\left\{\frac{1+2+3+\ldots+n}{2n^2}\right\}$ converges to

A. 0 B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. 1 E. NOTA

3. A ball dropped from a height of h feet is known to rebound to a height of ph feet where p is a positive constant less than one. The total distance traveled by the ball if it is dropped initially from a height of k feet is

A. $\frac{k}{1-p}$ B. $\frac{p}{1-p}$ C. $\frac{k-pk}{1-p}$ D. $\frac{k+pk}{1-p}$ E. NOTA

4. The value of $\sum_{n=1}^{5}\left(\frac{n}{2n-1}\right)^{(-1)^{(n+1)}}$, correct to the nearest integer is

A. -5 B. -4 C. 4 D. 5 E. NOTA

5. In a given arithmetic sequence, the first term is 1 and the sum of the first three terms equals the sum of the first ten terms. The sixth term of this sequence is

A. $\frac{-1}{3}$ B. $\frac{-1}{6}$ C. 0 D. $\frac{1}{6}$ E. NOTA

6. There are exactly two Fibonacci numbers, F_n, such that $F_n = n^2$. The larger of these two numbers is

A. F_{12} B. F_{14} C. F_{16} D. F_{18} E. NOTA

7. If the sum of n terms of a series is $a + bn + cn^2$, then the nth term, for $n > 1$, is

A. $a + b + 2^n c$ B. $b + 2nc$ C. $bn + cn$ D. $b+(2n-1)c$ E.NOTA

8. Consider the integers 175 and 2000. The sum of the arithmetic mean, the positive geometric mean, and the harmonic mean, to the nearest integer is

A. 1998 B. 1999 C. 2000 D. 2001 E. NOTA

9. $\sum_{n=1}^{2000} (-1)^n n^2 =$

A. 2,000,100 B. 2,001,000 C. 2,010,000 D. 2,100,000 E. NOTA

10. First express the infinite repeating decimal 0.234234234... as a reduced fraction. The sum of the numerator and the denominator of this fraction is

A. 99 B. 137 C. 411 D. 619 E. NOTA

11. For $i = \sqrt{-1}$, $\sum_{n=0}^{2000} i^n =$

A. -1 B. -i C. 1 D. i E. NOTA

12. Which of the following statements about infinite series is/are true?

I. If a series does not converge then it diverges.
II. A geometric series converges under the condition that the ratio r is less than one.
III. If the nth term of a series tends to zero as n tends to infinity, then the series converges.
IV. All arithmetic series diverge.

A. I. and II only B. II and III only C. I and IV only D. III. and V only E. NOTA

13. The sum of the first 50 odd positive integers is

A. 2000 B. 2100 C. 2300 D. 2400 E. NOTA

14. The greatest lower bound of the sequence $\left\{n^2 - 14n + 80\right\}$ is

A. 31 B. 40 C. 56 D. 67 E. NOTA

15. If a and b are positive integers, then $\sum_{k=0}^{\infty} \frac{a^k + b^k}{(a+b)^k}$ can be simplified to

A. 1 B. 2 C. $\frac{(a+b)^2}{ab}$ D. $a^2 + ab + b^2$ E. NOTA

16. $\sum_{n=0}^{100} \sin \frac{n\pi}{4} =$

A. $\frac{-\sqrt{2}}{2}$ B. 0 C. $1+\sqrt{2}$ D. $2+\sqrt{2}$ E. NOTA

17. Let R equal the limiting value obtained when the ratio test is applied to $\sum_{n=1}^{\infty} \frac{2^n}{n^3+1}$. $|R| =$

A. 0 B. $\frac{1}{2}$ C. 1 D. 2 E. NOTA

18. When $\sum_{n=1}^{2000} \ln\left(\frac{n}{n+1}\right)$ is expanded and combined, the value to the nearest hundredth is

A. -5.30 B. -5.60 C. -7.60 D. -7.89 E. NOTA

19. The sum of the firt n terms of the series whose nth term is $n^3 + \frac{n}{2}$ is

A. $\frac{5n^2+n}{4}$ B. $\frac{n^3+n}{4}$ C. $\frac{n(n^2+n+1)}{4}$ D. $\frac{n(n+1)(n^2+n+1)}{4}$ E. NOTA

20. If the roots of the cubic equation $x^3 - 12x^2 + 23x + c = 0$ form an arithmetic sequence, then the value of c is

A. -36 B. -34 C. 34 D. 36 E. NOTA

21. Which of the following is/are true

I. If an infinite series diverges, it diverges to negative infinity or to positive infinity.
II. If a finite number of terms are removed from an infinite divergent series, the series still still diverges
III. The nth term of a divergent series must be greater than one.
IV. The ratio of the (n+1) term to the n th term of a divergent series is always greater than one.

A. I only B. II only C. II and IV only D. II, III and IV only E. NOTA

22. $(1+x)^n =$

A. $\sum_{j=0}^{n} \binom{n}{j} x^j$ B. $\sum_{j=1}^{n} \binom{n}{j} x^j$ C. $\sum_{j=1}^{n+1} \binom{n}{j} x^{j-1}$ D. $\sum_{j=1}^{n-1} \binom{n}{j} x^{j+1}$ E. NOTA

23. If $2^{14} = 2 + \sum_{k=0}^{12} \log x^{2^k}$, where log x is in base 10, then x =

A. 2 B. 10 C. 100 D. 1,000 E. NOTA

24. The tens digit in the expansion of $\sum_{n=1}^{100} n!$ is

A. 1 B. 2 C. 4 D. 8 E. NOTA

25. If $\sum_{n=1}^{\infty} |a_n|$ converges then which of the following is/are true

I. $\sum_{n=1}^{\infty} a_n$ converges II. $\sum_{n=1}^{\infty} a_n$ is absolutely convergent III. $\sum_{n=1}^{\infty} -a_n$ converges

A. I only B. II only C. III only D. I and II only E. NOTA

26. $\sum_{n=1}^{\infty} \frac{n! + 2^n}{(2^n)(n!)} =$

A. e B. 1+ e C. 2 + e D. 2e E. NOTA

27. Let T_n represent the nth triangular number and S_n represent the nth square number.
For all n >1, $S_n - T_n =$

A. a prime number B. a triangular number C. a square number
D. a number divisible by 5 E. NOTA

28. $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots + (-1)^{n-1} \frac{x^{2(n-1)}}{(2(n-1))!} + \ldots$ is the Maclaurin series for

A. cos x B. sin x C. cosh x D. sinh x E.NOTA

29. The harmonic series can be proven divergent by using the

A. integral test B. derivative test C. ratio test D. root test E.NOTA

30. $\lim_{n \to \infty} \left(\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2 + n}} + \frac{1}{\sqrt{n^2 + 2n}} + \ldots + \frac{1}{\sqrt{n^2 + (n-1)n}} \right)$

can be evaluated by using the definite integral

A. $\int_0^1 \frac{1}{\sqrt{x}}\, dx$ B. $\int_1^2 \frac{1}{\sqrt{x}}\, dx$ C. $\int_0^2 \frac{1}{\sqrt{x}}\, dx$ D. $\int_1^{\infty} \frac{1}{\sqrt{x}}\, dx$ E.NOTA