6

NOTA means **"none of these answers"**

1. If i is the imaginary unit, then, $i + i^2 + i^3 + \ldots + i^n + \ldots + i^{1996} =$

A. $-i$ B. -1 C. 0 D. 1 E. NOTA

2. Consider the geometric series $1996^{10} + 1996^{20} + 1996^{30} + 1996^{40} + 1996^{50} + 1996^{60}$. The unit's digit of the sum of this series is:

A. 0 B. 2 C. 4 D. 6 E. NOTA

3. $\sum_{n=0}^{10} \left(\sin^2 n + \cos^2 n\right) =$

A. 0 B. 1 C. 10 D. 11 E. NOTA

4. The first three terms of a given geometric sequence are $\sqrt{2}$, $\sqrt[3]{2}$, $\sqrt[6]{2}$ .The sixth term of this sequence is:

A. 1 B. $\sqrt[-3]{2}$ C. $\sqrt[-6]{2}$ D. $\sqrt[-9]{2}$ E. NOTA

5. If $\sum_{n=1}^{\infty} \left(\frac{a}{b}\right)^n = \frac{1}{3}$, and a is an integer, then b is

A. one more than a multiple of 3
B. a multiple of 3
C. one more than a multiple of 4
D. a multiple of 4
E. NOTA

6. The seven-digit number that represents the sum of the first 1996 positive even integers contains each of the ten digits except:

A. 0, 2, and 3 B. 1, 5, and 7 C. 4, 6, and 7 D. 5, 6, and 7 E. NOTA

7. The infinite geometric series: $a + ar + ar^2 + \ldots + ar^n + \ldots$, has a finite sum if and only if

A. $r \neq 1$ B. $r \leq 1$ C. $|r| \leq 1$ D. $|r| < 1$ E. NOTA

8. If S is the sum of the first 1996 positive integers, which of the following statements, relative to S, is **not** true?

A. The middle digit is an even number.

B. The sum of the digits is a perfect number.

C. The left two digits are 1 and 9.

D. The unit's digit is a perfect number.

E. NOTA

9. Given the Fibonacci sequence: 1, 1, 2, 3, 5, . . . , where $F_1 = 1$, $F_2 = 1$ and for $n > 0$, $F_{n+2} = F_n + F_{n+1}$. Which of the following is/are true?

I. $F_{12} = 12^2$.
II. F_{16} is a prime number.
III. F_{96} is an odd number.

A. I only B. I and II only C. II and III only

D. I, II, and III E. NOTA

10. If a number is selected at random from the set of the first fifty Fibonacci numbers, the probability that that number is even is:

A. $\frac{8}{25}$ B. $\frac{17}{25}$ C. $\frac{37}{50}$ D. $\frac{39}{50}$ E. NOTA

11. Consider the arithmetic series: $k + \ldots + 92 + 94 + 96$. There exits two possible values of k which would make the sum of the series is 2340. The sum of these values is:

A. 2 B. 4 C. 6 D. 8 E. NOTA

12. The positive geometric mean of 499 and 1996 is:

A. divisible by 9 B. a perfect square C. an irrational number

D. greater than 1,000 E. NOTA

13. The measures of the seven angles of a convex heptagon are in arithmetic progression. If the measure of the smallest angle is 101° then the largest angle, to the nearest degree, has a measure of:

A. 108° B. 147° C. 156° D. 161° E. NOTA

14. $\sum_{n=1}^{10} (3n + 1) - \sum_{n=1}^{10} 3n + 1 =$

A. 0 B. 10 C. 11 D. 13 E. NOTA

15. An equilateral triangle has a side of length 12. Line segments join the midpoints of the sides to form a second triangle. Then, line segments join the midpoints of the sides of the second triangle to form a third triangle. If this process continues, what is the limiting sum of the perimeters of the triangles so formed?

A. 48 B. 72 C. 108 D. 144 E. NOTA

16. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} =$

A. e^{-1} B. π^{-1} C. $e - 2$ D. $\pi - 3$ E. NOTA

17. $\sum_{k=1}^{10} {}_{10}C_k =$

A. $2^9 - 1$ B. 2^{10} C. $2^{10} - 1$ D. $2^{10} + 1$ E. NOTA

18. A digital clock displays "1:23" and one notices that the three digits form an arithmetic sequence. How many times within the next hour are digits displayed in an arithmetic sequence?

A. 2 B. 3 C. 4 D. 5 E. NOTA

19. For the sequence $a_n = \frac{10^n}{n!}$, which of the following is/are true?

I. $a_9 = a_{10}$
II. $a_{n+1} \geq a_n$ for all n
III. $a_n \leq 3000$ for all n

A. I only B. I and II only C. I and III only D. I, II, and III E. NOTA

20. The series $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

A. converges to $\frac{e}{\pi - e}$ B. converges to $\frac{\pi}{\pi - e}$ C. converges to $\frac{1}{\pi - e}$

D. diverges E. NOTA

21. A ball is dropped from a height of n feet and continually rebounds 0.75 of the height it falls. What is the total distance that the ball can travel?

A. 2n feet B. 4n feet C. 7n feet D. 8n feet E. NOTA

22. Consider this program for the Sharp EL-9300C graphics calculator

```
n = 1
s = 0
Label 1
k = 2n -3
s = s + k
n = n + 1
If n < 9 Goto 1
Print s
```

When the program is executed, the output is:

A. 15 B. 20 C. 48 D. 80 E. NOTA

23. $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{k}{n^2} =$

A. 0.5 B. 1 C. e D. π E. NOTA

24. $\sum_{k=1}^{n} \log_2 k =$

A. $\log_2 n$ B. $\log_2 n^n$ C. $\log_2 (n+1)$

D. $\log_2 \frac{n(n+1)}{2}$ E. NOTA

25. If $\sum_{n=1}^{1996} n^3 = S$, then the number of digits in S is:

A. 10 B. 11 C. 12 D. 13 E. NOTA

26. Given two distinct positive integers, which of the following is/are true?

I. The arithmetic mean is always greater than the positive geometric mean.
II. The harmonic mean is always greater than the positive geometric mean.
III. The arithmetic mean is always greater than the harmonic mean.

A. I only B. II only C. I and III only D. I, II and III E. NOTA

27. If $[k]$ equals the greatest integer which does not exceed k, then $\sum_{n=1}^{21} \left[\sqrt{n}\right] =$

A. 46 B. 50 C. 54 D. 58 E. NOTA

28. $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}+\sqrt{n+1}} = k$. k is an integer if and only if:

A. n + 1 is a multiple of four B. n + 1 is a prime number C. n is a perfect square

D. n + 1 is a perfect square E. NOTA

29. Consider the equation: $x^3 + Ax^2 + Bx + C = 0$. If the three roots of the equation, when listed from smallest to largest, form an arithmetic sequence of positive integers then:

A. A is a multiple of 3 B. B is a multiple of 3 C. C is a multiple of 3

D. A, B, and C are each a multiple of 2 E. NOTA

30. Which of the following series is/are absolutely convergent?

I. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{(-1)^{n-1}}{n} + \ldots$

II. $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots + \frac{(-1)^{n-1}}{n^2} \ldots$

III. $1 - \frac{1}{2^1} + \frac{1}{2^2} - \frac{1}{2^3} + \ldots + \frac{(-1)^{n-1}}{2^{n-1}} + \ldots$

A. I only B. II and III only C. I and III only

D. I, II, and III E. NOTA