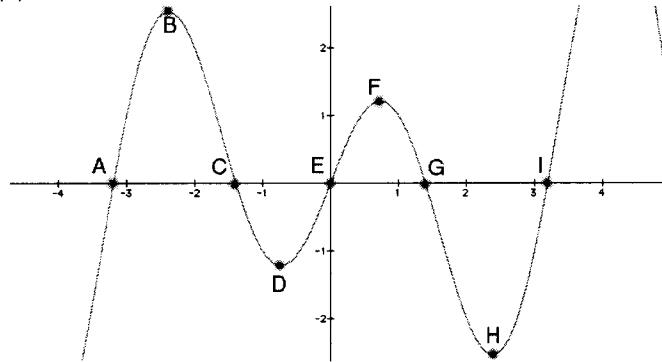


Question #1
Calculus Bowl
Mu Alpha Theta National Convention 2003

The graph below is a plot of $y = f'(x)$. How many of the statements below the graph are always true?



- I. A relative maximum of $f'(x)$ exists at point A.
- II. $f(x)$ is concave down at point B.
- III. $f(x)$ is concave up at point C.
- IV. $f''(x)$ at the point D is zero.
- V. $f(x)$ has an inflection point at point E.
- VI. A relative maximum of $f'(x)$ exists at point F.
- VII. A relative minimum of $f'(x)$ exists at point G.
- VIII. A relative minimum of $f'(x)$ exists at point I.

Question #2
Calculus Bowl
Mu Alpha Theta National Convention 2003

If $f(x) = (4x^2 + 1)^2(2x - 7)^3 \cos^3\left(x - \frac{\pi}{4}\right)$, then what is the value of $f'(0)$?

Question #3
Calculus Bowl
Mu Alpha Theta National Convention 2003

Let $A, B, C,$ and D (the constants in the following problems) all be natural numbers.

$$\int_1^A \sqrt{w-1} dw = 144$$

$$\int_{B/2}^B x^2 + 3x + 4 dx = 690$$

$$\int_1^C \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2} = \frac{2}{5}$$

$$\int_1^D \frac{(\log_5 z)^2}{z} dz = 9$$

What is the sum of $A, B, C,$ and D ?

Question #4
Calculus Bowl
Mu Alpha Theta National Convention 2003

At noon, the sailboat *Mu* was 12 nautical miles due north of the sailboat *Alpha*, whereas the sailboat *Theta* was 8 miles due east of the *Alpha*. The *Mu* was sailing south at 12 knots (nautical miles per hour) and continued to do so all day. The *Alpha* was sailing northeast at $8\sqrt{2}$ knots and continued to do so all day. The *Theta* was sailing west at 10 knots and continued to do so all day. Let A be the length of time (in minutes, to the nearest minute) when the *Mu* and the *Alpha* are closest together; let B be the length of time (in minutes, to the nearest minute) when the *Mu* and the *Theta* are closest together; and let C be the length of time (in minutes, to the nearest minute) when the *Alpha* and the *Theta* are closest together. What is the sum of $A, B,$ and C ?

Question #5
Calculus Bowl
Mu Alpha Theta National Convention 2003

Evaluate the following definite integral: $\int_{-4}^4 |x^3 - 7x - 6| dx$

Question #6
Calculus Bowl
Mu Alpha Theta National Convention 2003

This question is a relay type question. The answer to part (I), A , will be used in part (II)...and so on through part (III). **On your answer sheet, put down the exact value of C .**

(I). $\int_0^{2\pi} x \cos(x/2) dx = A$

(II). $\lim_{y \rightarrow 3} \frac{y^2 + 2y - 15}{y^2 - 8y + A + 23} = B$

(III). Let $f(z) = z^2 \cos(z/B)$. $f'(\pi) = C$

Question #7
Calculus Bowl
Mu Alpha Theta National Convention 2003

What is $A + 16B + \frac{C\sqrt{2}}{2} + 3D$ if...

$A = f(4)$ for $\int_0^w f(t) dt = w \cos(\pi w)$, $B = g(4)$ for $\int_0^x g(t) dt = x \cos(\pi x)$,

$C = h(2)$ for $h(y) = \frac{d}{dy} \left(\int_{-2y}^y t^3 \sqrt{4+td} \right)$, and $D = \lim_{z \rightarrow 0} \frac{\int_0^z \frac{t^2}{t^4+1} dt}{z^6}$?

Question #8
Calculus Bowl
Mu Alpha Theta National Convention 2003

A wire of length L is cut into two pieces, one being bent to form a square and the other to form an equilateral triangle. Let A be the length of the wire piece used to form the square if the sum of the two areas formed is a minimum. Let B be the length of the wire piece used to form the square if the sum of the two areas formed is a maximum. What is the sum of A and B ? (Note: Allow the possibility that the wire may not be cut at all, and only one shape is formed from the entire wire.)

Question #9
Calculus Bowl
Mu Alpha Theta National Convention 2003

What is $AB + CD$, if:

$A = \lim_{w \rightarrow 0} \tan 2w \csc 4w$

$B = \lim_{x \rightarrow 6} \frac{x^4 - 2x^3 - 23x^2 - 12x + 36}{x^4 - 6x^3 - 5x^2 - 106x + 120}$

$C = \lim_{y \rightarrow \infty} (3y^3 + 5)^{\frac{1}{\sqrt[3]{y}}}$

$D = \lim_{z \rightarrow \infty} \sqrt{z^2 - 5z} - \sqrt{z^2 + 7z}$

Question #10
Calculus Bowl
Mu Alpha Theta National Convention 2003

Of the four statements or expressions below, which one(s) is/are true? Mark the letters for the true responses on your answer sheet.

A. Calculating $\int_1^3 x^2 dx$ using the trapezoid rule with 6 subintervals on $[1,3]$ gives a value of $235/27$.

B. $\int_0^{\pi/4} \tan^4 x \sec^4 x dx = \frac{11}{35}$

C. The volume generated when the region bounded by the circle $y^2 + z^2 - 10y + 21 = 0$ in the yz -plane is revolved about the z -axis (to make a torus) is 40π .

D. The length of the curve $5y^3 = x^2$ that lies inside the circle $x^2 + y^2 = 6$ is $134/27$.

Question #11
Calculus Bowl

Mu Alpha Theta National Convention 2003

For each section below, let g be the inverse function of f .

(I.) $f(x) = 1 + 1/x$. $g'(3) = A$.

(II.) $f(x) = x^5 + x^3$. $g'(2) = B$.

(III.) $f(x) = x^2 - 4x - 3$, $x > 2$. $g'(2) = C$.

(IV.) $f(x) = \ln x$. $g'(1) = D$.

What is the value of $(1/A) + (1/B) + (1/C) + (1/D)$?

Question #12
Calculus Bowl

Mu Alpha Theta National Convention 2003

The curve whose derivative is $dy/dx = \sqrt{3y - 7x + xy - 21}$ passes through the points $(-2, 8)$, $(1, A)$, $(6, B)$, $(13, C)$, and $(22, D)$. What is the sum of A, B, C , and D ?

Question #13
Calculus Bowl

Mu Alpha Theta National Convention 2003

If...

$$A = \int_0^{\sqrt{3}} x^3 \sqrt{1+x^2} dx, \quad B = \int_0^{\sqrt{\pi/2}} x \tan^2(x^2) dx,$$

$$C = \int_3^5 \frac{3x^2 - 6x - 28}{x^3 - 3x^2 - 28x + 60} dx, \quad \text{and} \quad D = \int_1^e x(\ln x)^2 dx,$$

then what is the value of $A - 2B + e^C + D - \frac{58}{15}$?

Question #14
Calculus Bowl

Mu Alpha Theta National Convention 2003

The legs of an isosceles triangle have length 12 each. The vertex angle (the angle opposite the base) is decreasing at a rate of 0.5 radians per second. The parameter A is equal to the rate of change of the area of the triangle when the vertex angle is $\pi/3$ radians. The parameter B is equal to the rate of change of the perimeter of the triangle when the vertex angle is $\pi/3$ radians. What is $|A| + |B|$ (ignore the difference between length and area units)?

Question #15
Calculus Bowl

Mu Alpha Theta National Convention 2003

If $f(x) = \frac{4x}{\sqrt{8x^2 + 1}}$, then what is the value of the expression below?

$$\left[f(\sqrt{3}) \right]^2 + 25f'(-\sqrt{3}) + \lim_{x \rightarrow \infty} f(x) + \int_0^{\sqrt{3}} f(x) \sqrt{8x^2 + 1} dx$$