School Bowl Solutions - Mu Level
2000 Mu Alpha Theta National Convention

① $A = 0,\ B = 4;$

$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$ is the definition of derivative of $\sqrt{x}$, which is $\frac{1}{2\sqrt{x}}$.

At $x = 4$, $\frac{1}{2\sqrt{4}} = 1/4$.

② $\text{Area} = \int_1^4 x^2\,dx = \frac{x^3}{3}\Big]_1^4 = \frac{64}{3} - \frac{1}{3} = 21 \to A$

$T = \frac{4-1}{2\cdot 3}(1 + 2\cdot 4 + 2\cdot 9 + 16) = 43/2 \to B$

$f_{avg} = \frac{1}{4-1}\int_1^4 x^2\,dx = \frac{1}{3}(21) = 7 \to C$

$\frac{2AB}{C} = \frac{2(21)(43/2)}{7} = 129$

③ $f'(3) = \frac{3^2 g'(3) - g(3)\cdot 2\cdot 3}{3^4} = \frac{19}{27} \to A$

$h'(3) = 3^2 g'(3) + g(3)\cdot 2\cdot 3 = 33 \to B$

$k'(3) = 2\cdot 3 + g'(3) = 11 \to C$

$A\left(\frac{B}{C}\right)^3 = \frac{19}{27}(3)^3 = 19$

④ $\int_2^3 \frac{dx}{x^2 - x} = \int_2^3\left(\frac{1}{x-1} - \frac{1}{x}\right)dx = \left(\ln(x-1) - \ln x\right)\Big]_2^3$

$= (\ln 2 - \ln 3) - (\ln 1 - \ln 2) = \ln\frac{4}{3} \to A$

$\int_{-1}^2 |x^2 - 1|\,dx = \int_{-1}^1 (1 - x^2)\,dx + \int_1^2 (x^2 - 1)\,dx$

$= \left(x - \frac{x^3}{3}\right)\Big]_{-1}^1 + \left(\frac{x^3}{3} - x\right)\Big]_1^2 = \frac{8}{3} \to B$

$\int_1^3 x^{-2}\,dx = -x^{-1}\Big]_1^3 = -\frac{1}{3} + 1 = \frac{2}{3} \to C$

$\frac{e^A C}{B} = \frac{e^{\ln 4/3}\cdot 2/3}{8/3} = \frac{4/3 \cdot 2/3}{8/3} = \frac{1}{3}$

⑤ $6x + 4x^2\frac{dy}{dx} + 8xy + 2xy\frac{dy}{dx} + y^2 = 0$,

so $\frac{dy}{dx} = \frac{-6x - 8xy - y^2}{4x^2 + 2xy}$; at $(1,1)$, $\frac{dy}{dx} = \frac{-5}{2}$

Eqn of tangent line: $y - 1 = -\frac{5}{2}(x-1)$ or

$y = \frac{-5}{2}x + \frac{7}{2}$; so y-int is $7/2 \to A$

⑤ (cont.) Distance from $(x, \sqrt{3x})$ to $(5,0)$ is $d = \sqrt{(x-5)^2 + 3x}$; $d' = \frac{2x-7}{2\sqrt{x^2 - 7x + 25}}$

Minimum is at $x = 7/2 \to B$.

$A + B = 7/2 + 7/2 = 7$

⑥ $\int_0^2 x^2 e^{x/2}\,dx = e^{x/2}(2x^2 - 8x + 16)\Big]_0^2$

$= 8e - 16$

⑦ $V = s^3$; $\frac{dV}{dt} = 3s^2\frac{ds}{dt}$; $-4 = 3(25)\frac{ds}{dt}$

so $\frac{ds}{dt} = \frac{-4}{75}$. $A = 6s^2$; $\frac{dA}{dt} = 12s\frac{ds}{dt}$

$= 12(5)(-4/75) = -16/5 \to A$

For $y = \frac{(x-2)^2}{x+1}$, vertical asymptote occurs at $x = -1$. Since $\frac{(x-2)^2}{x+1}$

$= x - 5 + \frac{9}{x+1}$, oblique asymptote has equation $y = x - 5$; so $B = -1, C = 1, D = -5$

$|A|BCD = \frac{16}{5}(-1)(1)(-5) = 16$

⑧ $(x+4)^2 + (y-3)^2 = 4$; distance from $(-4,3)$ to line $x = 2$ is 6.

Area of circle is 4π. $V = 2\pi r\cdot\text{Area}$

$= 2\pi(6)(4\pi) = 48\pi^2 \to A$

$V = \pi\int_0^2 (8x - x^4)\,dx = \pi\left(4x^2 - \frac{1}{5}x^5\right)\Big]_0^2$

$= \pi\left(16 - 32/5\right) = \frac{48\pi}{5} \to B$

$A/B = 48\pi^2\cdot\frac{5}{48\pi} = 5\pi$

⑨ $f(1) = \int_1^1 \sin(\ln 2t)\,dt = 0$

$f'(x) = \sin(\ln 2x)$ so $f'(.5e^\pi) = \sin\pi = 0$

$f''(x) = \frac{\cos(\ln 2x)}{x}$ so $f''(.5) = \frac{\cos 0}{.5} = 2$

$f'''(x) = \frac{-\sin(\ln 2x) - \cos(\ln 2x)}{x^2}$

so $f'''(.5) = \frac{-\sin 0 - \cos 0}{(.5)^2} = -4$

$0 + 0 + 2 + (-4) = -2$

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(10) Given $\frac{db}{dt} = 2000e^{5t/6}$; $\int db =$ $2000\int e^{5t/6}dt$; $b = 2400e^{5t/6} + C$; (condition yields $c=0$). When $t=12$, $b = 2400e^{5\cdot 12/6} = 2400e^{10}$

(11) $y' = 3x^2 - 6x - 9$; $y'' = 6x - 6 = 0$ at $x=1$

$y'(1) = 3 - 6 - 9 = -12 \rightarrow A$

Area $= s^2$; $2s\frac{ds}{dt} = 3\frac{ds}{dt}$; $s = \frac{3}{2} \rightarrow B$

Since function is decreasing on $1 \le x \le 2$, minimum occurs at $x=2$;

$2^2 - 5\cdot 2 + 4 = -2 \rightarrow C$

Particle changes direction when $v(t) = 10 - 8t = 0$; $t = 5/4$.

$s(1) = 6$
$s(5/4) = 6.25$
$s(2) = 4$

$> .25$
> 2.25

$2.5 \rightarrow D$

$AB + CD = -12\cdot\frac{3}{2} + (-2)(2.5)$

$= -18 + -5 = -23$

(12) $\int_0^c \left(x - \frac{1}{c}x^2\right)dx = \left(\frac{x^2}{2} - \frac{x^3}{3c}\right)\Big]_0^c$

$= \frac{c^2}{2} - \frac{c^3}{3c} = \frac{c^2}{6} = 54 \Rightarrow C = 18$

(13) $\frac{df}{dx} = \frac{x^2}{2} - x + 1$ and

$f = \frac{x^3}{6} - \frac{x^2}{2} + x - \frac{2}{3}$

$f(-1) = \frac{-1}{6} - \frac{1}{2} - 1 - \frac{2}{3} = -\frac{7}{3}$

(14) $A = (1, \infty)$.

$\lim_{n\to\infty} \left|\frac{(k-1)^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n\cdot 2^n}{(k-1)^n}\right|$

$= \left|\frac{k-1}{2}\right| < 1$ for $-1 < k < 3$;

include $k = -1$; so $B = [-1, 3)$

$A \cap B = (1, 3)$

(15) $\int_0^{\pi/4} \tan^4 x\,dx = \int_0^{\pi/4} \tan^2 x(\sec^2 x - 1)dx$

$= \int_0^{\pi/4} \tan^2 x \sec^2 x\,dx - \int_0^{\pi/4}(\sec^2 x - 1)dx$

$= \left(\frac{\tan^3 x}{3} - \tan x + x\right)\Big]_0^{\pi/4} = \frac{1}{3} - 1 + \frac{\pi}{4}$

$= \frac{\pi}{4} - \frac{2}{3} \rightarrow A$

$\int_0^{\pi/2} \cos^3 x\,dx = \int_0^{\pi/2} \cos x(1 - \sin^2 x)\,dx$

$= \int_0^{\pi/2} \cos x\,dx - \int_0^{\pi/2} \sin^2 x \cos x\,dx$

$= \left(\sin x - \frac{\sin^3 x}{3}\right)\Big]_0^{\pi/2} = 1 - \frac{1}{3} = \frac{2}{3} \rightarrow$

$A + B = \frac{\pi}{4} - \frac{2}{3} + \frac{2}{3} = \frac{\pi}{4}$