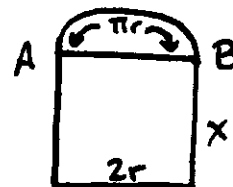


1989 NATIONAL MATH
RELATED RATES, MAXIMA, MINIMA TEST

- 1) The figure to the right consists of a semicircle with a rectangle constructed on its diameter. If the perimeter of the figure is 20', then the conditions for maximum area is: (Arc $AB = \pi r$).

a) $x = \frac{\pi}{r}$ b) $x = r$ c) $x = 2r$ d) $x = \frac{r}{2}$ e) nota



- 2) A conical reservoir has a depth of 24' and a circular top of 12' (radius). It is being filled so that the depth of water is increasing at a constant rate of 4'/hr. Determine the rate in cubic ft/hr at which the water is entering the reservoir when the depth is 5'.

a) 16π b) 25π c) 75π d) 100π e) nota

- 3) The total area of the surface of a cylinder (including ends) is 150π ft². The volume is a maximum when:

a) $r = 3'$ b) $r = 2\pi$ ft. c) $r = \frac{\pi}{2}$ ft. d) $r = 5'$ e) nota

- 4) A light is on top of a pole h ft. high. A ball is dropped from a point at the same height as the light but k feet horizontally away from it. How fast is the shadow of the ball moving along the ground $\frac{1}{2}$ second later? (Assume the ball falls a distance of $16t^2$ ft. in t seconds.)

a) $-kh$ b) kh c) $-2kh$ d) $2kh$ e) nota

- 5) The curve $y = x^2 - 1$ is revolved about the y -axis to form a container. If liquid flows into the container at a rate of 2 cu. units/min., how fast is the depth of the liquid changing when the depth of the liquid is 5 units?

a) $\frac{5\pi}{2}$ b) $\frac{2}{5\pi}$ c) $\frac{1}{2\pi}$ d) 2π e) nota

- 6) A farmer estimates that if he digs his potatoes now, he will have 160 bushels worth \$2/bushel. If he waits, the crop will increase by 40 bushels/week, but the selling price will drop 20¢ per bushel, per week and picking costs will go up 5¢ per bushel, per week. When should he dig his potatoes to get the most income?
 a) 2 days b) 5 days c) 1 week d) 2 weeks e) nota
- 7) A piece of wire 60 inches long is cut into 6 sections, 2 of one length and 4 of another length. Each of the two sections having the same length is bent into the form of a circle and the two circles are then joined by the four remaining sections to make a frame for a model of a right circular cylinder. Find the length of the radius which will make the cylinder of maximum volume.
 a) $\frac{\pi}{10}$ b) $\frac{\pi}{2}$ c) $\frac{10}{\pi}$ d) $\frac{2}{\pi}$ e) nota
- 8) A straight fence 100 yds. long stands on a ranch. The fence is to be left standing, and part or all of it is to be used in forming a rectangular corral, using an additional 260 yds. of fencing for the other 3 sides. Find the maximum area which can be so enclosed.
 a) 8450 sq.yd. b) 6400 sq.yds. c) 8000 sq.yds. d) 10,000 sq.yds. e) nota
- 9) A rectangular sheet of paper containing 1,000 square inches is to hold printed matter in a section having a one-inch margin on each side, a two-inch margin at the top, and a three-inch margin at the bottom. What should the dimensions of the sheet of paper be if the printed matter is to have a maximum area?
 a) 50x20 b) 100x10 c) 25x40 d) 125x8 e) nota
- 10) Two ships start at the same point, but the first ship leaves at noon and sails east at the rate of 20mph and the second leaves at 1pm and sails south at 30mph. At 2pm, how fast is the distance between them changing?
 a) $50\sqrt{2}$ b) $40\sqrt{2}$ c) $35\sqrt{2}$ d) $25\sqrt{2}$ e) nota

- 11) The radius r of a sphere is increasing at the uniform rate of .3" per second. At the instant when the surface area S becomes 100π sq.in., what is the rate of increase, in cubic inches per second, in the volume V ?
- a) 10π b) 12π c) 22.5π d) 25π e) nota
- 12) A cylindrical can is undergoing a transformation in which the radius and height are varying continuously with time t . The radius is increasing at 4in/min, while the height is decreasing at 10in/min. At what rate is the volume changing when the radius is 3" and the height is 5"?
- a) 210π b) 45π c) 39π d) 30π e) nota
- 13) A frugal young man has decided to extract one of his teeth by tying a stout rubber band to a chain on a garage door opener which runs on a horizontal track 3' above his mouth. If the garage door opener moves the chain at $\frac{1}{4}$ ft/sec, how fast is the rubber band expanding when it is attached to a length of 5'?
- a) $\frac{1}{5}$ fps b) $\frac{1}{4}$ fps c) $\frac{1}{3}$ fps d) $\frac{1}{2}$ fps e) nota
- 14) When a stone is dropped in a pool, a circular wave moves out from the point of impact at a rate of 6 in/sec. How fast is the area enclosed by the wave increasing when the wave is 2" in radius?
- a) 4π b) 12π c) 24π d) 48π e) nota
- 15) A chord of a circle of radius r is revolved about a line tangent to the circle and parallel to the chord so as to form a cylindrical band. Find the maximum area such a band can have.
- a) $\frac{r}{2}$ b) $3\pi r^2\sqrt{3}$ c) $4\pi r^2\sqrt{3}$ d) $\pi r^2\sqrt{3}$ e) nota

- 16) A barge, whose deck is 10' below the level of a dock, is being drawn in by means of a cable attached to the deck and passing through a ring on the dock. When the barge is 24' away, and approaching the dock at $\frac{3}{4}$ ft. per second, how fast is the cable being pulled in? (Neglect any sag in the cable.)
- a) $\frac{9}{13}$ fps b) $\frac{5}{13}$ fps c) $\frac{12}{13}$ fps d) $\frac{9}{26}$ fps e) nota
- 17) Two parallel sides of a rectangle are being lengthened at the rate of 2 in/sec, while the other 2 sides are shortened in such a way that the figure remains a rectangle with constant area A of 50 square inches. What is the rate of change of the perimeter P when the length of an increasing side is 5"?
- a) -4in/sec b) 4in/sec c) -12in/sec d) 12in/sec e) nota
- 18) Sand falling from a chute forms a conical pile whose altitude is always equal to $\frac{4}{3}$ the radius of the base. How fast is the radius increasing when it is 6' and the volume is increasing at the rate of $24 \text{ ft}^3/\text{min}$?
- a) $\frac{1}{9\pi}$ ft/min b) $\frac{1}{4\pi}$ ft/min c) $\frac{1}{3\pi}$ ft/min d) $\frac{1}{2\pi}$ ft/min e) nota
- 19) Find the radius of the base of the right circular cone of minimum volume which can be circumscribed about a sphere of radius 8".
- a) 24 b) 32 c) $8\sqrt{2}$ d) 64 e) nota
- 20) The cost of fuel in running a locomotive is proportional to the square of the speed and is \$25 per hour for a speed of 25mph. Other costs amount to \$100 per hour, regardless of the speed. Find the speed which will make the cost per mile a minimum.
- a) 25mph b) 45mph c) 50mph d) 60mph 3) nota

- 21) A drawbridge with two 10' spans is being raised at a rate of 2 radians/minute. How fast is the distance increasing between the ends of the spans when they both are at an elevation of $\pi/4$ radians?
- a) $5\sqrt{2}$ b) $10\sqrt{2}$ c) $20\sqrt{2}$ d) $30\sqrt{2}$ e) nota
- 22) A revolving beacon located 30 miles from a straight shore line makes 2 revolutions/min. Find the speed of the spot of light along the shore when it is two miles away from the point on the shore nearest the light.
- a) $\frac{13}{3}\pi$ b) 4π c) 13π d) $\frac{52}{3}\pi$ e) nota
- 23) A wire 24" long is cut in two, and then one part is bent into the shape of a circle and the other into the shape of a square. How should it be cut if the sum of the areas of the circle and the square is to be a minimum?
- a) $\frac{24\pi}{4+\pi}; \frac{96}{4+\pi}$ b) $\frac{96\pi}{4+\pi}; \frac{24}{4+\pi}$ c) $\frac{8\pi}{4+\pi}; \frac{32}{4+\pi}$ d) $\frac{32\pi}{4+\pi}; \frac{8}{4+\pi}$
- e) nota
- 24) The current, I (in amperes) in a certain electrical circuit is given by $I = \frac{100}{R}$, where R denotes resistance (in ohms). Find the rate of change of I with respect to R when the resistance is 20ohms.
- a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) $\frac{1}{4}$ d) $-\frac{1}{4}$ e) nota
- 25) A cube whose edge is x is contracting. When its surface area is changing at a rate which is equal to 6 times the rate of change of its edge, then the length of the edge is:
- a) 2 b) $3/4$ c) 1 d) $4/3$ e) nota

- 26) An airplane is flying west at 500 ft/sec. at an altitude of 4000'. The airplane is in a vertical plane with a searchlight on the ground. If the light is to be kept on the plane, how fast is the searchlight revolving when the airplane is due east of the searchlight at an airline distance of 2000'.
- a) $\frac{1}{2}$ rad/sec b) $\frac{1}{3}$ rad/sec c) $\frac{1}{5}$ rad/sec d) $\frac{1}{10}$ rad/sec
e) nota
- 27) The demand equation for a particular shirt is $2px + 65p - 4950 = 0$, where x hundreds of shirts are demanded per week when p dollars is the price of the shirt. If the shirt is selling this week at \$30 and the price is increasing at the rate of \$.20 per week, find the rate of change in the demand.
- a) $-\frac{11}{20}$ shirt/week b) $\frac{11}{20}$ shirt/week c) -55 shirt/week
d) 55 shirt/week e) nota
- 28) For a package to be accepted by a particular mailing service, the sum of the length and girth (perimeter of a cross section) must not be greater than 100". If a package is to be in the shape of a rectangular box with a square cross section, find the dimensions of the package having the greatest possible volume that can be mailed by the service.
- a) $\frac{100}{3} \times \frac{50}{3} \times \frac{50}{3}$ b) $100 \times \frac{50}{3} \times \frac{50}{3}$ c) $\frac{100}{3} \times 50 \times 50$
d) $100 \times 50 \times 50$ e) nota
- 29) A man 6' tall is walking toward a building at the rate of 4fps. If there is a light on the ground 40' from the building, how fast is the man's shadow on the building changing when he is 30' from the building?
- a) -8.4 b) 8.4 c) -9.6 d) 9.6 e) nota

30) Boyle's Law for the expansion of gas is $PV=C$, where P is the pounds per square unit of pressure, V is the number of cubic units of volume of the gas, and C is a constant. At a certain instant the pressure is 3000 lb/ft^2 , the volume is 5 ft^3 , and the volume is increasing at the rate of $3 \text{ ft}^3/\text{min}$. Find the rate of change of the pressure at this instant.

- a) 1800 lb/ft^2 b) -1800 lb/ft^2 c) 600 lb/ft^2
d) -600 lb/ft^2 e) nota