

Mu Alpha Theta National Convention: Seattle, 1997
Calculus Probability and Statistics Topic Test

1. Express as an ordered triplet the mean, mode, and median of the following data set:
5, 8, 4, 2, 1, 4, 13, 7, 9, 4, 9
(A) (6,5,4) (B) (4,5,9) (C) (6,9,5) (D) (6,4,5) (E) NOTA

2. A card is drawn from a deck. What is the probability that it is a prime number or a heart?
(A) $\frac{23}{52}$ (B) $\frac{6}{13}$ (C) $\frac{25}{52}$ (D) $\frac{1}{2}$ (E) NOTA

3. My science fiction collection consists of 8 books by Larry Niven, 4 by Orson Scott Card, and 3 by Carl Sagan. If I wish to keep books by the same author together, in how many ways can I arrange the books upon a shelf?
(A) 2,903,040 (B) 5,806,080 (C) 34,836,480 (D) 69,672,960 (E) NOTA

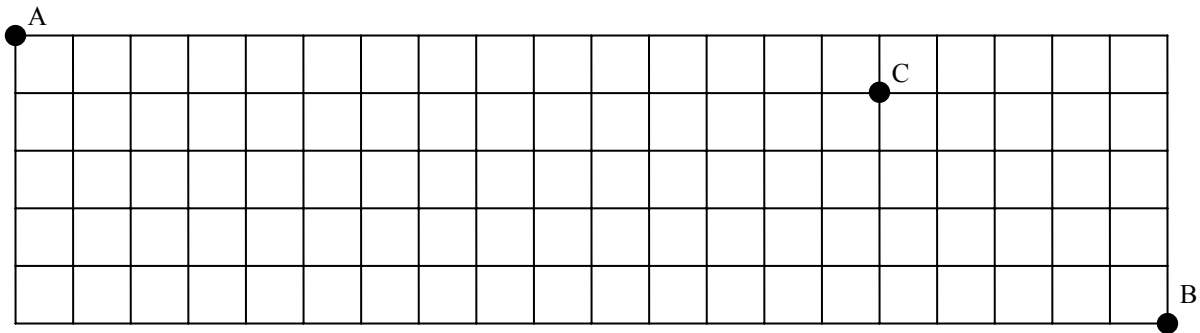
4. Two coins are drawn from a bag which contains 8 quarters, 5 dimes, and 3 nickels. What is the probability that the total amount of money drawn is less than 35¢?
(A) $\frac{13}{60}$ (B) $\frac{7}{30}$ (C) $\frac{1}{4}$ (D) $\frac{4}{15}$ (E) NOTA

5. A student guesses at each of 40 true/false questions. What is the probability he gets at least 20 correct?
(A) $\frac{1}{2}$ (B) $1 - \frac{\binom{40}{20}}{2^{40}}$ (C) $\frac{1}{2} + \frac{\binom{40}{20}}{2^{41}}$ (D) $\frac{1}{2} + \frac{\binom{40}{20}}{2^{40}}$ (E) NOTA

6. In a horse race, the odds are 14:1 against 'Smiling Doug' placing. In another horse race, the odds are 20:1 against 'Kev' placing. What is the probability that neither horse places?
(A) $\frac{19}{21}$ (B) $\frac{8}{9}$ (C) $\frac{31}{35}$ (D) $\frac{41}{45}$ (E) NOTA

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7. A person travels from point A to point B always moving either down or right. What is the probability they go through point C?



- (A) $\frac{63}{1265}$ (B) $\frac{72}{1265}$ (C) $\frac{81}{1265}$ (D) $\frac{90}{1265}$ (E) NOTA
8. In how many ways can 5 students seat themselves in 12 desks?
- (A) 79380 (B) 84240 (C) 88720 (D) 95040 (E) NOTA
9. A coin is flipped ten times. What is the probability that it comes up heads at least 4 times in a row?
- (A) $\frac{249}{2^{10}}$ (B) $\frac{251}{2^{10}}$ (C) $\frac{253}{2^{10}}$ (D) $\frac{255}{2^{10}}$ (E) NOTA
10. The probability of event A is $\frac{5}{6}$. The probability of event B, given that event A occurs, is $\frac{2}{3}$. The probability of event A, given that event B occurs, is $\frac{7}{9}$. What is the probability that neither event occurs?
- (A) $\frac{1}{378}$ (B) $\frac{2}{378}$ (C) $\frac{1}{252}$ (D) $\frac{1}{126}$ (E) NOTA
11. What is the coefficient of the fifth term in the expansion of $(-x^3 + 3z)^{11}$?
- (A) -26730 (B) 38824 (C) -74038 (D) 112266 (E) NOTA

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12. Three dice are rolled. At least one of them shows a 6. What is the probability that at least one of them shows a 2?
- (A) $\frac{91}{216}$ (B) $\frac{1}{3}$ (C) $\frac{30}{91}$ (D) $\frac{36}{91}$ (E) NOTA
13. Consider the data set: 5, 8, 19, 17, 27, 12, 6, x, y, where x equals the mean of the data set and y is the median of the data set. What is the maximum sum of x and y?
- (A) $\frac{92}{3}$ (B) 31 (C) $\frac{97}{3}$ (D) 33 (E) NOTA
14. Four distinct numbers are chosen from the first nine natural numbers. What is the probability that 5 is the second largest of those chosen?
- (A) $\frac{4}{21}$ (B) $\frac{5}{21}$ (C) $\frac{10}{63}$ (D) $\frac{16}{63}$ (E) NOTA
15. A certain math competition pits two teams against one another in a series of rounds until one of the teams has beaten the other in five of the rounds. In how many ways can the competition transpire if no round can result in a tie?
- (A) 252 (B) 256 (C) 500 (D) 512 (E) NOTA
16. How many distinct orders are possible for 9 keys arranged on a key ring?
- (A) 13440 (B) 20160 (C) 40320 (D) 392880 (E) NOTA
17. The scores on this test will exhibit a normal distribution with a mean of 90 and a standard deviation of 10. What percentage (to the nearest tenth of a percent) of the competitors will receive a score greater than 110? Use the attached table.
- (A) 2.3 (B) 2.5 (C) 2.6 (D) 2.9 (E) NOTA
18. Consider two bags of marbles. Bag A contains 6 blue marbles and 2 red marbles. Bag B contains 5 red marbles and 6 blue ones. A marble is taken from Bag A and placed in Bag B, then a marble, which happens to be blue, is drawn from Bag B. What is the probability that the marble drawn from Bag A was blue?
- (A) $\frac{27}{48}$ (B) $\frac{1}{8}$ (C) $\frac{21}{27}$ (D) $\frac{7}{16}$ (E) NOTA

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19. Three dice are rolled. What is the expected value of the sum of the larger two? I.e. the die with the lowest value is ignored, and the other two are summed.
- (A) $\frac{203}{24}$ (B) $\frac{1831}{216}$ (C) $\frac{69}{8}$ (D) $\frac{611}{72}$ (E) NOTA
20. How many students take math and science, but not history, if, in a school with 214 students, 156 students take math, 122 students take science, 88 take history, 75 take both science and math, 34 take history and science, and 6 take all three?
- (A) 47 (B) 58 (C) 69 (D) cannot be determined (E) NOTA
21. Jon will arrive home from work sometime between 5 and 6 PM, stay for five minutes, then leave to have dinner. Joy will call Jon sometime between 5 and 6 PM, let the phone ring for 2 minutes, then give up. What is the probability that the phone will be ringing while Jon is home?
- (A) $\frac{811}{7200}$ (B) $\frac{203}{1800}$ (C) $\frac{31}{300}$ (D) $\frac{7}{60}$ (E) NOTA
22. What is the probability that when three points are chosen at random on the circumference of a circle, it is possible to draw a diameter of the circle which has all three points on the same side of the diameter?
- (A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{5}{8}$ (D) $\frac{3}{8}$ (E) NOTA
23. What is the sum of the determinants of all possible 3 by 3 matrices formed using the first nine natural numbers exactly once?
- (A) 0 (B) 1 (C) 9 (D) 9^9 (E) NOTA
24. 5 numbers are selected from the first 200 natural numbers. What's the probability that they are drawn in increasing order?
- (A) $\frac{5}{676}$ (B) $\frac{3}{400}$ (C) $\frac{1}{120}$ (D) $\frac{2}{125}$ (E) NOTA

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25. The scores on this test will follow a normal distribution with mean 90 and standard deviation 10. What percentage of the competitors (to the nearest tenth of a percent) will receive scores between 70 and 80? Use the attached chart.

- (A) 12.1 (B) 13.6 (C) 14.2 (D) 17.7 (E) NOTA

26. The probability of number B being less than x is $p_B(x) = \frac{x^2}{16}$ for $0 \leq x \leq 4$, is 0 for $x < 0$, and 1 for $x > 4$. What is the probability that B is 1?

- (A) $\frac{1}{16}$ (B) $\frac{3}{32}$ (C) $\frac{1}{8}$ (D) $\frac{5}{32}$ (E) NOTA

27. What is the expected value of the number Q, if the probability that Q is less than x is

$$p_Q(x) = 1 - \frac{(x-4)^2}{16} \text{ for } 0 \leq x \leq 4, \text{ is 0 for } x < 0, \text{ and 1 for } x > 4.$$

- (A) 1 (B) $\frac{7}{6}$ (C) $\frac{4}{3}$ (D) $\frac{3}{2}$ (E) NOTA

28. Imagine “cloudiness” to be a real number between zero and one. For a given cloudiness, C , the probability that it rains is $p_R(C) = \frac{C^2}{4} + \frac{C}{2}$. In Seattle in August, the probability that it is less cloudy than x is $p_C(x) = 1 - (x-1)^4$. What is the probability that it rains on a given day in Seattle in August?

- (A) $\frac{13}{120}$ (B) $\frac{7}{60}$ (C) $\frac{1}{8}$ (D) $\frac{2}{15}$ (E) NOTA

29. Two points are chosen on the circumference of a circle. What is the probability that the chord whose endpoints are those two points is longer than the radius of the circle?

- (A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) 1 (E) NOTA

30. The probability that R is greater than x is $P_R(x) = \frac{(x-2)^3}{16} + \frac{1}{2}$ for $0 \leq x \leq 4$, is 1 for $x < 0$, and 0 for $x > 4$. What is the expected value of the square of R?

- (A) $\frac{83}{15}$ (B) $\frac{94}{15}$ (C) $\frac{104}{15}$ (D) $\frac{112}{15}$ (E) NOTA