

2002 National Mu Alpha Theta Convention
Mu Division—Number Theory Topic Test

1. How many primes are there between 100 and 120?
A. 5 B. 6 C. 7 D. 8 E. NOTA

2. What is the greatest common factor of $5!$, $15!$, and $25!$?
A. $5!$ B. $25!$ C. $15!$ D. 5 E. NOTA

3. How many positive integer divisors does 121 have?
A. 4 B. 3 C. 2 D. 1 E. NOTA

4. If m and n are positive integers such that $m^2 + 49 = n^2$, then what are all possible values of $n - m$?
A. 1 or 3 B. 1 or 7 C. 1, 7, or 49 D. 1 E. NOTA

5. Given three integers $i, j, k > 1$ such that i is a perfect square, j a cube, and k a fourth power, then the product ijk must have which of the following:
I. An integer as a sixth root.
II. An integer as an eighth root.
III. An integer as a twelfth root.
IV. An integer as a twenty-fourth root.
A. I. only B. I. and III. only C. I., II., and III. only D. All four E. NOTA

6. If a and b are nonnegative integers and 2^a divides 3^b , then which of the following must be true?
A. $a = 0$ B. $b = 0$ C. $a \neq 0$ D. $b \neq 0$ E. NOTA

7. How many perfect squares fall between 20^3 and 21^3 ?
A. 6 B. 7 C. 8 D. 9 E. NOTA

8. How many positive integer pairs (m, n) satisfy the equation $m^2 = 4n + 3$?
A. 7 B. 1 C. 0 D. infinitely many E. NOTA

9. What is the largest prime less than 1000?
A. 991 B. 997 C. 983 D. 989 E. NOTA

10. What is the least common multiple of 4^4 , $4!$, and 44 ?
A. 8448 B. 270336 C. 16896 D. 33792 E. NOTA

2002 National Mu Alpha Theta Convention
Mu Division—Number Theory Topic Test

11. Find the smallest positive integer n such that $7n$ leaves a remainder of 3 when divided by 5.

- A. 1 B. 2 C. 3 D. 4 E. NOTA

12. Find the sum of all positive integers less than 200 which have 9 distinct divisors.

- A. 332 B. 136 C. 296 D. 232 E. NOTA

13. How many positive divisors does $9!$ have?

- A. 80 B. 16 C. 512 D. 160 E. NOTA

14. What is the largest positive integer that cannot be expressed in the form $5n + 7m$ for some pair of positive integers n and m ?

- A. 23 B. 35 C. 47 D. 58 E. NOTA

15. Order from least to greatest:

$$x = 2^{2^{2^2}}, \quad y = 3^{3^{3^3}}, \quad z = 4^{4^4}.$$

- A. x, y, z B. y, z, x C. z, x, y D. x, z, y E. NOTA

16. What is the largest value of n such that $2002!$ is evenly divisible by 20^n ?

- A. 1995 B. 1321 C. 500 D. 289 E. NOTA

17. How many pairs of positive integers (m, n) satisfy

$$m^3 + 63n^3 = 777700006?$$

- A. 12 B. 1 C. 0 D. infinitely many E. NOTA

18. The nonnegative integer n is a quadratic residue (mod m) if there is an integer k such that $k^2 \equiv n \pmod{m}$. How many nonnegative integers less than 103 are quadratic residues (mod 103)?

- A. 52 B. 51 C. 50 D. 53 E. NOTA

19. How many integers less than one million have initial digit 6 and have the property that if this leading 6 is deleted, the remaining integer is $1/25$ the original number?

- A. 64 B. 16 C. 4 D. 1 E. NOTA

20. How many four-digit integers are perfect squares and have only even digits?

- A. 3 B. 4 C. 5 D. 6 E. NOTA

2002 National Mu Alpha Theta Convention
Mu Division–Number Theory Topic Test

21. How many pairs of integers (m, n) satisfy the equation

$$\frac{1}{m} + \frac{1}{n} = \frac{1}{10}?$$

A. 17 B. 18 C. 8 D. 9 E. NOTA

22. What is the tens digit of 7^{7^7} ?

A. 0 B. 2 C. 4 D. 6 E. NOTA

23. The 9th and 10th grade chess tournament has n total students. There are 10 times as many 10th graders as 9th graders in the tournament. Every student plays every other student once, and in each game either the winner is awarded one point and the loser none or the competitors draw and each gets $1/2$ point. If the 10th graders together amass only 4.5 times as many points as the ninth graders, what are the possible positive values of n ?

A. 11 or 22 B. 11 C. 22 D. any positive integer divisible by 11 E. NOTA

24. For how many positive integers m less than 1000 is $m^{3^{10}-3^9} - 1$ divisible by 3^{10} ?

A. 333 B. 666 C. 999 D. 1 E. NOTA

25. *What is the sum of the digits of the sum of the digits of the sum of the digits of 4444^{4444} ?

A. 25 B. 16 C. 11 D. 7 E. NOTA

*=Tie-breaker