2

1999 MAΘ National Convention Number Theory Mu Division

1. In a certain base, b, 2 times the number represented by the numeral 137 is 5 times the number represented by the numeral 51. Write 25_{10} in base b.

A. 7 B. 9 C. 23 D. 27 E. 46

2. If m = the number of positive integral divisors of 2700, and n = the sum of the positive integral divisors of 2700, find n+m.

A. 20 B. 36 C. 234 D. 8680 E. 8716

3. A child's stacking toy is made of 10 different plastic cubes with side lengths 1 cm, 2 cm, 3cm, . . . 10 cm. Instead of stacking them, Bobby decides to pour his grape juice in them. To the nearest tenth, how many 2 liters of grape juice will the toys hold?

A. 1.1 B. 1.5 C. 2.0 D. 2.5 E. 3.0

4. For all integers, m, if 2m - 3 is divisible by 8, which of the following is also divisible by 8?

A. 2m + 3 B. 2m - 8 C. 4m + 8 D. 5m + 1 E. 6m - 9

5. If $A = \sum_{k=3}^{45} k!$, then what is the remainder when A is divided by 240?

A. 144 B. 150 C. 152 D. 170 E. 220

6. How many prime numbers between 50 and 100 have remainder 3 when divided by 6?

A. 0 B. 2 C. 5 D. 8 E. 11

7. How many 3 digit numbers have at least one 2 or one 3 as digits?

A. 448 B. 452 C. 504 D. 648 E. 900

8. For all integral values of m, $m^2 + a$ is not divisible by 10. If a is a single-digit positive number, find the sum of all possible values of a.

A. 2 B. 5 C. 12 D. 15 E. 20

9. What is the remainder when $(19^{19} + 99^{99})$ is divided by 7?

A. 2 B. 3 C. 4 D. 5 E. 6

10. Using digits from the set {1, 2, 3, 4, 5} without repetition, how many 3-digit multiples of 11 can be formed?

A. 2 B. 4 C. 8 D. 12 E. 24

11. If m and n are a set of twin primes such that 18 more than twice m is equal 7 less than 3 times n, find $m^2 - n^2$.

A. 37 B. 72 C. 80 D. 120 E. 220

12. Find the sum of the x values of the positive integral pairs of solutions to the equation $4x + 3y = 38$?

A. none B. 8 C. 10 D. 13 E. 15

13. Find the 18^{th} term in the Fibonacci - like sequence 1,2,3,5,8,13,...

A. 19 B. 171 C. 1597 D. 2584 E. 4181

14. The difference between the squares of 2 positive integers cannot have a remainder m when divided by 8. Which of the following is a possible value of m?

A. 0 B. 1 C. 3 D. 4 E. none of these

15. What is the remainder when 1999^{1999} is divided by 19?

A. 0 B. 4 C. 5 D. 6 E. 9

16. Which of the following has the same remainder when divided by 4, 5, and 9?

A. 253 B. 287 C. 321 D. 357 E. 722

17. Suppose m is an integer such that $m(9!) = \frac{151!}{142!}$ Find the largest prime divisor of m.

A. 5 B. 7 C. 13 D. 29 E. none of these

18. Find the next term in the sequence: 3, 5, 9, 17, 32, 58, . . .

A. 48 B. 84 C. 95 D. 100 E. 116

19. How many zeros are at the end of $(22!)^2$ when it's written in base 4?

A. 11 B. 19 C. 22 D. 38 E. 44

20. Simplify completely: $[\{A\cap(B\cup C)\}\cap\{A\cup(B\cap C)\}]\cap(C\cup B^C)$

A. $A\cap B\cap C$ B. $A\cup B$ C. $A\cap B$ D. $A\cap C$ E. $\varnothing$

21. Which of the following is divisible by 3 for all integral values of n?

A. n^3-7n+3 B. n^3-8n+3 C. n^2-1 D. n^3-2n E. n^3-3n

22. A triangular pyramid of ping pong balls is being formed so that each ball rests in the space of the 3 below it. Only the bottom n levels have been completed, using 185 balls. Find n.

A. 3 B. 4 C. 5 D. 6 E. 7

23. If $25\equiv 85 \bmod m$ and $52\equiv 92 \bmod m$, then m must divide which of the following?

A. 2 B. 5 C. 10 D. 20 E. 25

24. How many integral pairs (x,y) are solutions to the following system of equations:
$(x-1)^2-(y-3)^2=0$ and $\left|\frac{1}{3}x+y\right|<3$.

A. 0 B. 11 C. 13 D. 17 E. ∞ many

25. If you choose a number from the set 11-30 inclusive, and x is the probability you've chosen a deficient number, which of the following describes x?

A. $x<0.2$ B. $0.2\le x<0.4$ C. $0.4\le x<0.55$ D. $0.55\le x<0.7$ E. $0.7\le x<0.85$

26. Theorem: For all sets A and B, $A^c\cup B^c\subseteq(A\cup B)^c$
Proof: Suppose A and B are sets, $x\in A^c\cup B^c$

a. $x\in A^c$ or $x\in B^c$
b. $x\notin A$ or $x\notin B$
c. $x\notin A\cup B$
d. $x\in(A\cup B)^c$
e. $A^c\cup B^c\subseteq(A\cup B)^c$

In which step of the proof is there an error in logic?

A. a B. b C. c D. d E. e

27. Let m = the largest integral divisor of k^3-k where k is any odd integer. Let n= the largest integral divisor of p^3-p where p is any even integer. Find $\left|\frac{m-n}{2}\right|$.

A. 1 B. 1.5 C. 6 D. 9 E. 18

28. When simplified completely, $\frac{1}{(2!)(4!)}+\frac{1}{(3!)(3!)}=\frac{m}{n}$, where m and n are integers and relatively prime, find the least common multiple of m and n.

A. 24 B. 144 C. 151 D. 1008 E. 2016

29. Given $x^2 - dy^2 = 1$ for d= 7, 11, or 18, find the smallest positive integral value of y that results in a solution to the equation. What is the sum of these y values?

A. none B. 3 C. 4 D. 6 E. 10

30. If $\sum_{n=2}^{8} a\int_{1}^{11} x^n dx$ is evaluated, the result is a positive integer. What is the smallest possible value of the positive integral constant, a?

A. 3 B. 7 C. 21 D. 63 E. none of these

Tie Breaker 1: What is the fourth largest Mersenne prime?

Tie Breaker 2: On a given sheet of 8 ½ x 11 paper, n lines are drawn. No three lines are concurrent, and no two lines are parallel. If n=1, there are two regions formed. Find the number of regions formed for n=5, 10, and 100. What is the sum of the number of regions for these three cases?

Tie Breaker 3: How many Pythagorean triples of the form $m^2 - n^2$, $2mn$, $m^2 + n^2$ have all three integers between 100 and 200 and include at least one multiple of 7?