

# True Number Theory

- ✓ ⑩ 132      253      and reverses      8
- 143
- 154

⑪  $18 + 2m = 3n - 7$   
 $13 + 2(n+2) = 3n - 7$       or       $18 + 2(n-2) = 3n - 7$   
 $18 + 2n + 4 = 3n - 7$        $18 + 2n - 4 = 3n - 7$   
 $22 + 2n = 3n - 7$        ~~$2k = n$~~  not prime

29 = n       $m^2 - n^2$   
31 = n+2      = (m-n)(m+n)  
    = 2(60) = 120      D

⑫  $4x + 3y = 38$

$x = 8 - 3t$        $y = 2 + 4t$

	0	1	2		
x = 8 - 3t	8	5	2	-1	15
y = 2 + 4t	2	6	10		13

E

- ⑬ 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181      E

- ⑭ 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

$8k+1$        $8k+4$        $8k$        $8k+3$

~~A~~      ~~B~~      ~~C~~      ~~D~~      E

$8M-8N$        $8M+1-8N$        $8M+4-8N$

⑮  $9 \overline{) 1999}$  R4       $1999^1 \equiv 4 \pmod{19}$        $9 \overline{) 1999}$  R1

$1999^2 \equiv -3 \pmod{19}$        $19 \overline{) 769}$  R8

$1999^3 \equiv 7 \pmod{19}$        $\frac{152}{17}$

$1999^5 \equiv -2 \pmod{19}$        $1999 = 1999 \cdot 1$

$1999^9 \equiv 1 \pmod{19}$        $1999 \cdot 4 \pmod{19} = 1979$

$9 \overline{) 1999}$  R1

$19 \overline{) 1999}$  R18

B

B I

B III

C I

D II

E IIII

## Number Theory - mu

$$\textcircled{1} \quad 2(b^2 + 3b + 7) = 5(5b + 1) \quad b = 9 \quad 25_{10} = 27_9 \quad \boxed{D}$$

$$2b^2 + 6b + 14 = 25b + 5$$

$$2b^2 - 19b + 9 = 0$$

$$(2b - 1)(b - 9) = 0$$

$$\textcircled{2} \quad 2700 = 2^2 \cdot 5^2 \cdot 3^3 \quad \# \text{ of divisors } 3 \cdot 3 \cdot 4 = 36$$

sum of divisors  $(1+2+4)(1+5+25)(1+3+9+27)$

$$(7)(31)(40) = 8680 + 36 = 8716 \quad \boxed{E}$$

$$\textcircled{3} \quad 1^3 + \dots + 10^3 = \left[ \frac{10(11)}{2} \right]^2 = 3025 \text{ cc} = 3025 \text{ ml}$$

$$= 3.025 \text{ l} \sim 1.5 \text{ liters} \quad \boxed{B}$$

$$\textcircled{4} \quad 2m - 3 \mid 3 = 6m - 9 \quad \boxed{E}$$

$$\textcircled{5} \quad 3! + 4! + 5! + 6! + \dots$$

$$6 + 24 + 120 (+720 + \dots) = 150 + 240k - 150 \quad \boxed{B}$$

$$\textcircled{6} \quad 6k + 3 = 3(2k + 1) \text{ not prime unless } k = 0$$

not in range 50-100 zero  $\boxed{A}$

$$\textcircled{7} \quad \text{digits } 100 - 99,9 \quad 900 - 7.8.8$$

$$452 \quad \boxed{B}$$

$$\begin{array}{r} 2 \\ 64 \\ 7 \\ \hline 448 \end{array}$$

$$\textcircled{8} \quad m^2 = 1 \quad 4 \quad 9 \quad 16 \quad 25 \quad 36 \quad 49 \quad 64 \quad 81 \quad 100$$

$$\text{not } a \quad 9 \quad 6 \quad 1 \quad 4 \quad 5 \quad 4 \quad 1 \quad 6 \quad 9$$

$$a \quad 2, 3, 7, 8 \quad 20 \quad \boxed{E}$$

$$\textcircled{9} \quad 19 \equiv 5 \pmod{7}$$

$$99 \equiv 1 \pmod{7}$$

$$19^2 \equiv 4 \pmod{7}$$

$$99^{11} \equiv 1 \pmod{7}$$

$$19^3 \equiv -1 \pmod{7}$$

$$19^5 \equiv 3 \pmod{7}$$

ANSWER : 6

$$19^{10} \equiv 2 \pmod{7}$$

$$19^9 \equiv -1 \pmod{7}$$

$$19^{19} \equiv -2 \pmod{7}$$

$$\boxed{E}$$

$$\begin{array}{r} 277 \\ 7 \overline{) 1902} \\ \underline{140} \\ 502 \\ \underline{490} \\ 12 \\ 39 \\ \underline{35} \\ 4 \end{array}$$

16. must be multiple of 4, 5, + 9 + 1, 2, 3

LCM =  $36 \cdot 5 = 180, 360, 720, 540$

**E**

17.  $m \cdot 9! = 151 \cdot 150 \cdot 149 \cdot 148 \cdot 147 \cdot 146 \cdot 145 \cdot 144 \cdot 143$

largest prime factor of  $m = 151$

**E**

18. 3, 5, 9, 17, 32, 58, 100

$\sqrt{2}, \sqrt{4}, \sqrt{8}, \sqrt{15}, \sqrt{26}, 42$

$\sqrt{2}, \sqrt{4}, \sqrt{7}, \sqrt{11}, \sqrt{16}$

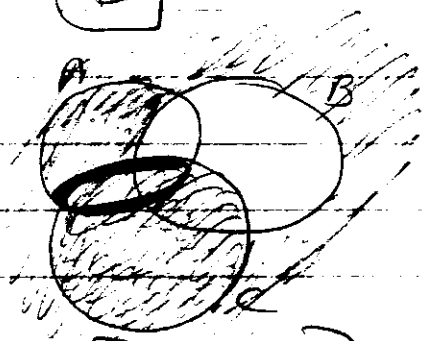
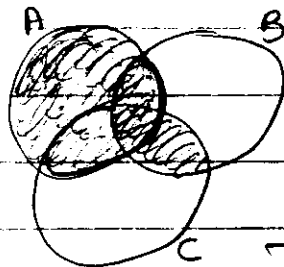
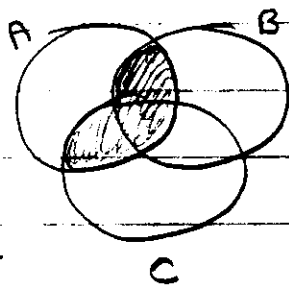
$\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}$

**D**

19.  $(22!)^2 = (11 + 5 + 2 + 1) \cdot 2$

**D**

20



$[A \cap (B \cup C)] \cap [A \cup (B \cap C)] \cap [C \cup B^c]$

**D**

21.  $n^3 - 7n + 3$

$n^3 - n - 6n + 3$

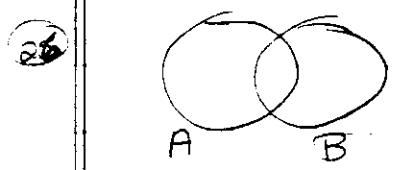
**A**

22. same as 29 alpha

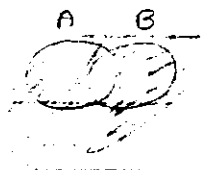
23 Same as 28 alpha

25 Same as 27 alpha

24 Same as 30 alpha



$x \in A$  or  $x \in B$   
 $x \in A \cup B$  error  
C



27  $K(K-1)(K+1)$   $K$  odd something  $\div$  by 3  
 $\div 4$   
 $\div 2$

24

$K$  even  $K(K-1)(K+1)$  something  $\div 2$   
 something  $\div 3$

6  
D

28  $\frac{1}{2(24)} + \frac{1}{36} = \frac{m}{n}$

$$\frac{1}{4 \cdot 12} + \frac{1}{3 \cdot 12} =$$

$$\frac{3+4}{3 \cdot 4 \cdot 12} = \frac{7}{144}$$

$$\frac{7}{1008}$$

D

$\frac{3}{72}$        $\frac{7}{102}$        $\frac{13}{108}$   
 $\frac{13}{72}$        $\frac{7}{102}$        $\frac{13}{108}$   
 $\frac{180}{282}$

29.  $x^2 - dy^2 = 1$

$x^2 = dy^2 + 1$

$x^2 = 7y^2 + 1$

$y = 3$

$x^2 = 11y^2 + 1$

$= 3$

$x^2 = 18y^2 + 1$

$y = 4$

**E**

30.  $\sum_{n=2}^8 a \int_1^{11} x^n dx$

$\sum_{n=2}^8 a \frac{x^{n+1}}{n+1} \Big|_1^{11}$

$a \left[ \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} + \frac{x^6}{6} + \frac{x^7}{7} + \frac{x^8}{8} + \frac{x^9}{9} \right]_1^{11}$

$\left[ \frac{11^3-1}{3} + \frac{11^4-1}{4} + \frac{11^5-1}{5} + \frac{11^6-1}{6} + \frac{11^7-1}{7} + \frac{11^8-1}{8} + \frac{11^9-1}{9} \right]$

$\frac{14641}{14641}$   
 $\frac{161051}{161051}$   
 $\frac{1771561}{1771561}$   
 $\frac{19487171}{19487171}$   
 $\frac{214358881}{214358881}$   
 $\frac{2357947691}{2357947691}$

$\frac{1331-1}{3}$        $\frac{14641-1}{4}$        $\frac{161050}{5}$        $\frac{1771560}{6}$        $\frac{19487170}{7}$        $\frac{2357947690}{9}$

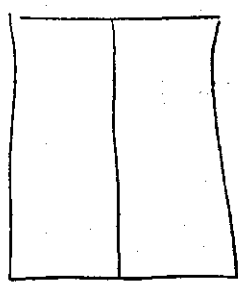
$\frac{1330}{3}$        $\frac{14640}{4}$        $2783881$

**3**      reduces      reduces      reduces      **7**      reduces      **9**

**63**      **D**

TB1  $2^2 - 1 = 3$      $2^3 - 1 = 7$      $2^5 - 1 = 32$      $2^7 - 1 = 127$

TB2



line  
region  
similar to

1	2	3	4	5	6	7
2	4	7	11	16	22	29

$\frac{n(n+1)}{2}$  but +1       $\frac{n(n+1)}{2} + 1$

regions =  $\frac{n^2 + n}{2} + 1$

$n=5$        $\frac{25+5}{2} + 1 = 16$

$n=10$        $\frac{100+10}{2} + 1 = 56$

$n=100$        $\frac{10000+100}{2} + 1 = 5051$

+  
5051  
56  
16  
**5123**

TB3 See earlier sheet